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An Assessment of Pre-Service Mathematics Teachers' Techno-Pedagogical Content Knowledge regarding Geometry

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ABSTRACT

Despite the significance of incorporating technology into geometry instruction, many teachers still find it difficult to teach geometry using technology. This study employed a techno- pedagogical content knowledge (TPACK) survey of 70 pre-service mathematics teachers (29 males, 41 females) from a public university. The goal was to determine which domains, if any, were open for development so that a course could be designed to meet these needs. The data was analysed thematically according to the TPACK sub-domains. The results showed that the pre-service teachers were confident in the areas of pedagogy knowledge, content knowledge, and pedagogical content knowledge (90% confidence) and open to improvement in the areas of technology, technological content, and technological pedagogy knowledge (70% confidence). As a result, we suggest creating a 14-week course to increase pre-service mathematics teachers' TPACK of these components in the hopes of bridging the knowledge gap identified in this study.

Keywords:
Geometry teaching, pre-service mathematics teachers, survey research, TPACK

1. Introduction

Teachers are one of the key factors in students' learning; they shape the future of their students. Research has argued that teacher knowledge affects how they teach and how students learn (Shulman, 1986; 1987). Although there is an ongoing debate about whether or not teacher knowledge can or should be categorised, teacher knowledge is still categorised in order to improve in many fields (Mishra & Koehler, 2006; Niess, 2005, 2011). Geometry research has consistently revealed a lack of teacher knowledge in teaching geometry with technology (Ball et al., 2008; Mishra & Koehler, 2006; Niess, 2008; Saralar-Aras, 2022; Young et al., 2019). Indeed, The UN Agenda stressed that by 2030, the goal is substantially increasing the supply of qualified teachers in terms of values, knowledge, and skills (The United Nations, 2015). Teaching with and through technology has become more popular with the COVID-19 pandemic. Given this lack of knowledge, it is critical to improve lessons to assist future teachers in improving their technology, pedagogy, and content knowledge (shortly TPCK or TPACK).

TPACK is a framework Mishra and Koehler (2006) designed to evaluate teachers' knowledge. It started as a technology extension of Shulman's (1986) pedagogical content knowledge (PCK). TPACK framework has seven sub-knowledge domains, each emphasizing different knowledge and corresponding skills for teachers. These components are technology knowledge (TK), pedagogy knowledge (PK), content knowledge (CK), pedagogical content knowledge (PCK), techno-pedagogical knowledge (TPK), technological content

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knowledge (TCK), and finally techno-pedagogical content knowledge (TPACK). It is important to state the definitions of each of these components for this study.

- Pedagogy knowledge (PK) is "the knowledge about the processes and practices or methods of teaching and learning" (Koehler & Mishra, 2009, p.64). It is regarding the issues on understanding teaching and learning processes, techniques, and approaches.
- Technology knowledge (TK) is "the knowledge about standard technologies, such as books, chalk and blackboard, and more advanced technologies, such as the Internet and digital video. In the case of digital technologies, this includes knowledge of operating systems and computer hardware, and the ability to use standard software tools such as word processors, spreadsheets, browsers, and e-mail" (Mishra and Koehler, 2006, p. 1027). It is about the knowledge of basic and advanced technology, such as books, chalk, and blackboards, as well as the Internet and digital video. This involves understanding operating systems and computer hardware and the ability to utilise common software tools such as word processors, spreadsheets, browsers, and e-mail in the case of digital technologies
- Technological pedagogy knowledge (TPK) is "the knowledge of the existence, components, and capabilities of various technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as the result of using particular technologies" (Mishra & Koehler, 2006, p.1028). It is the knowledge of the presence, elements, and potential of various technologies as they are utilised in teaching and learning contexts, as well as how teaching could change due to employing certain technologies.
- Content knowledge (CK) is "subject matter knowledge of a teacher to be learned or taught" (Koehler & Mishra, 2009, p.63). It is the subject area expertise of a teacher to acquire or explain to others.
- Pedagogical content knowledge (PCK) is "the content knowledge that deals with the process, including how the subject matter is presented and formulated to make it understandable to others" (Shulman, 1986, p.9). It covers issues related to the subject knowledge that concerns the teaching process, such as how to describe and formulate the subject so that others can understand it.
- Technological content knowledge (TCK) is "the knowledge about how technology and content are reciprocally related" (Mishra & Koehler, 2006, p.1028). In other words, it includes the knowledge concerning how technology and content are mutually connected and affect each other.
- Technological pedagogical content knowledge (TPACK) is related to the types of knowledge (4.1 to 4.6) teachers must have to successfully integrate technology into the classroom

Today, TPACK is an integral part of the education system, as it incorporates the increasing need for technology in the classroom, and the continued emphasis on curriculum and how we teach it. It then creates education for the future and the establishment of pre-service teachers (PSTs) for their future.
Geometry is a branch of mathematics which studies the properties and relations of points, lines, surfaces, solids, and higher dimensional analogues (Saralar-Aras, 2022). Learning mathematics is an essential competency for students for their future lives because of various reasons including working in occupations that require maths such as accounting and mathematics teaching and, basic life requirements like paying for a bill (Bottge, 1999; Gutstein, 2003; Lochhead & Whimbey, 1987; Skemp, 1987). To illustrate, mathematics, specifically geometry, has a lot in common with natural sciences, where natural scientists use geometric knowledge and skills; thus, it can be used by many disciplines, including architecture, biology, chemistry, physics, engineering, geology, and medicine (Skemp, 1987). Geometry has always been regarded as an essential topic for study (Clements, 2003; Lindquist & Clements, 2001; Tahta, 1980; Tutak & Adams, 2015). Fractal geometry, for example, has an important impact for architecture. The sample review demonstrates that architecture is not designed to be isolated but to anticipate changes in the surroundings using geometry (Lu et al., 2012). Interpretations of plane geometry also have been widely used in biology and genetics for ultrasound and MRI scans, which are vital for our lives (Gu et al., 2016; Haji et al., 2016).

Although there are many motivations for teaching geometry, there are also many variables that influence teaching this discipline, such as human factors (e.g., ability and motivation), social factors (e.g., gender and socioeconomic status), and pedagogical/instructional factors (e.g., curriculum policies and teaching staff) (Saralar-Aras & Esen, 2021). Researchers (e.g., Darling-Hammond, 2006; Hew & Brush, 2007; Sahlberg, 2012; Smith et al., 2016; Wright et al., 1997) report that teachers are the most influential element in students' success through their teaching activities. They all suggested that it is worth researching the principles and priorities of teachers, since teaching could be defined as an engaging experience between students and teachers, and teaching involves the understanding and thinking of teachers, hence teachers' perceived knowledge domains are important. It is widely known that TPACK is context-bound (Mishra & Koehler, 2006), a teacher's or a pre-service teacher's gender (e.g., Koh & Chai, 2011), age (e.g., Lin et al., 2013) or grade level (e.g., Jang, 2021) have all been found to be relevant constructs for their perception of TPACK. Despite the fact that related literature has shown that gender is a non-significant variable (e.g., Koh & Chai, 2011; Schmid et al., 2021; Wang, 2022), how PSTs' perceptions differ based on their grade levels and teaching and course experiences (e.g., courses related to mathematics teaching methods and educational technologies) opened up a new world of wonder for us (Saralar-Aras, 2022; Scherer et al., 2018; Schmid et al., 2021).

Learning pre-service teachers' conceptions on their TPACK is valuable for future teachers. Particularly, in the researched context, there needs to be a course designed to close the gap in pre-service teachers' TPACK.

Moreover, technology is an important part of our lives. Future teachers will be required to teach with or through technology, especially considering the COVID-19 pandemic. According to Voogt and McKenney (2017) basic hardware and software knowledge can be called basic technology literacy, which is the technology knowledge in the TPACK framework. When these issues are handled in the wider context of TPACK, knowing the distinction between hardware and software determines teachers' use of technology: the tools to prioritise and integrate effectively into the necessary steps of the lessons (Voogt & McKenny, 2017), which is associated with technological content knowledge (Mishra & Koehler, 2006). The distinction between hardware and software is seen as valuable in the context of early literacy.

This study looked at pre-service mathematics teachers' techno-pedagogical content knowledge with a special interest in researching technology integration into geometry teaching. The results of the study will feed into the design of an elective course for pre-service teachers to improve their techno-pedagogical content knowledge. The longer-term goal of this work is to improve pre-service mathematics teachers' techno-pedagogical content knowledge by giving them the opportunity to practice teaching through the designed elective course in the coming cycles, so that pre-service teachers can better cope with classroom problems in the future by having the necessary TPACK knowledge.

1.1. TPACK Studies in Mathematics Education (and Geometry in Particular)

Research evidence shows that TPACK studies in the related literature are mainly conducted by adding other mathematics topics such as ratio and proportion rather than geometry specific (see Za'ba et al., 2020). Furthermore, when we checked the significant databases establishing the journals from high impact factors (e.g., Web of Science), the sample geometry specific TPACK studies from the last five years came up with the following in Table 1.

Our rationale for studying pre-service mathematics teachers on their TPACK, particularly in geometry topics, is threefold. First, in the research of teacher education, the research design of the studies is frequently in quantitative methodologies (50%), then qualitative (37%), and instructional design (13%). We realised the quantitative approach to TPACK in geometry is preferred in the related literature. Second, the study groups were found to be frequent (50%) with pre-service mathematics teachers. However, in terms of geometrical topics, the studies seemed to fluctuate in that very few explicitly mentioned specific topics such as polygons or quadrilaterals. The studies tend to construct their methodology on geometrical cognition tools such as GeoGebra or Geometer's SketchPad, the effect of instructional designs. Instead of TPACK knowledge, beliefs, and from the views of pre-service teachers, efficacy levels, the research did not point out TPACK dimensions deeply grounded by Mishra and Koehler (2006). We came up with the notion that literature review in TPACK studies, particularly in geometry education, has been currently missing the determination of PSTs' needs on which dimension of TPACK skills. Then, we could design an instruction, apply a treatment or do other educational analyses. With these ideas in mind, we sought an answer to the perceived TPACK levels and sub-levels of pre-service mathematics teachers in their third and fourth (senior) years, and whether there is any gender difference in these scores.

Table 1. *Limited Number of TPACK Studies*

Researchers	Research design	Study group	Geometry topics	Dependent variables
Abunda (2021)	Qualitative (Instructional design)	Maths teachers	Coordinate geometry	Development of teacher module and its evaluation
Galanti et al. (2021)	Mixed (Survey design and end of term reflection)	Primary school teachers, coaches and specialists from multiple K-12 school	Use of dynamic geometry tools such as GeoGebra and Cabri for geometry content	Maths learning trajectory
Zambak & Tyminski (2020)	Qualitative (Exploratory case study)	Pre-service maths teachers	Use of dynamic geometry tools: GeoGebra or Geometer's SketchPad in general	Development of an assessment rubric to measure PSTs geometry MKT for teacher educators
Açıkgül & Aslaner (2020)	Quantitative (True Experiment as 2 x 2 factorial design)	Maths teachers	Polygons	TPACK efficacy and self-efficacy perception levels
Açıkgül & Aslaner (2019)	Quantitative (Correlation study)	Pre-service maths teachers	Polygons	TPACK efficacy level and self-efficacy perception
Saralar et al. (2018)	Qualitative (Descriptive case study)	Pre-service maths teachers	3D shapes	TPACK knowledge during practicum
Alizadeh-Jamal et al. (2018)	Quantitative (Experimental design with pre- post-lesson survey)	Maths teachers	Polygons, circle, and coordinate systems	TPACK knowledge and belief
Farrarrello et al. (2017)	Quantitative (Experimental design)	High school teachers and their students	Quadrilaterals	Overall assessment of the course, self-assessment of skills and personal comments

1.2. The Significance of the Study

Mathematics, particularly geometry, is one of the fields in the world where students' success in national and international exams is lower than expected (Saralar-Aras & Esen, 2021). Even adults believe they are not successful enough in basic mathematical literacy in their daily lives. Some of the reasons for this include seeing the fields as an abstract concept; the intensity of curricula; inability to use in problem-solving, inadequate use of diverse teaching methods; insufficient use of field-specific teaching approaches (Larkin & Jorgensen, 2016; Saralar-Aras, 2022).

It is seen that K-12 students may have mathematical anxiety, and hence attitudinal and emotional constructs towards those fields (Barroso et al., 2021; Dowker et al., 2016; Frenzel et al., 2007; Lee & Koo, 2020). Now, with the exponential development of technology, the fact that technology literacy does not progress at a similar pace, the lack of knowledge, skills and anxieties in the use of technological tools continue to increase this under achievement and inequality in educational opportunity. In teacher education, it is seen that most teacher educators transfer their teaching approach to pre-service teachers and so the pre-service teachers tend to use these approaches in their teaching in real classrooms. These pre-service teachers start to teach the way they were taught (Birgili et al., 2016; Saralar et al., 2018). Studies by Van Petegem et al. (2005), and Cancino et al. (2020) show that pre-service teachers use these methods and approaches in their teaching in the same way as they were taught to integrate technology, pedagogy and content knowledge and skills in the lessons of their faculties of education. However, as Santos and Castro (2021) noted: "Using technology may change the way teachers teach" (p. 2). What makes this process successful is not learning how technology is taught but using technology as an effective tool in the courses. That is to say, studies stress that pre-service teachers to learn how technology can help to enhance students' learning in mathematics instead of learning how to teach technology.

As previously noted, a few review studies highlight the TPACK model and its usefulness (Koehler & Mishra, 2009; Niess, 2005, 2008) and adoptions of TPACK to different disciplines, including science and foreign language teaching (Kaur et al., 2021; Li, 2021; Shin et al., 2021). Some discuss the importance of TPACK on in-service teachers' professional development including studies on these teachers' perceived TPACK (Harris & Hofer, 2009; Harris, 2016). But the development of a field does not depend only on theoretical development. TPACK as a field of study is progressing with scientific results by collecting data from the field with multi-dimensional and evidence-based approaches. In those areas, not only quantitative (e.g., Koh & Chai, 2014) but also qualitative (e.g., Huang et al., 2021) and mixed methods (e.g., Hwang & Lajoie, 2021; Kaya & Elster, 2019) studies have been implemented with pre-service, in-service teachers and even experts such as teacher educators (e.g., Voogt & McKenney, 2017). Up to the current date, the number of studies conducted with pre-service teachers (e.g., Balgalmış, 2013; Mourlam et al., 2021; Thohir et al., 2020) seems to be slightly higher than those conducted with in-service teachers (e.g., Choi & Young, 2021; Harris & Hofer, 2009; Harris, 2016). Since, although teachers gain experience in content knowledge and pedagogical knowledge over the years, it has been discussed in studies that increasing the skills of teachers in educational technologies while they are novices allows them to improve their TPACK in the following years. For example, Jang and Tsai (2012) studied with in-service elementary mathematics and science teachers in Taiwan. They investigated how to enhance in-service teachers' ability to utilize educational technology, focusing on the relation between TPACK and their use of interactive whiteboards (IWB). They were specifically interested in gender and subject area differences between teachers. The teachers were asked to complete an IWB-TPACK survey developed by Jang and Tsai (2012). Interestingly, the study results showed that the year of seniority in the field is a significant variable for the teaching and learning process; varying years of teaching experience affect the efficiency and effectiveness of the teaching and learning process, hence teachers' TPACK. For this reason, it is important to study the development of technology, pedagogy, and content knowledge with the participants who have similar levels of knowledge, and if there are pre-service teachers who do not have well-developed technology, pedagogy, and content knowledge as yet, these probably are a good fit for such research to evaluate their technology, pedagogy, and content knowledge and develop instructional designs for the improvement of their TPACK, accordingly. Last but not least, although many studies used surveys as data collection tools, they mostly analysed pre-service teachers in terms of different descriptive variables in the context of TPACK. In addition, these studies revealed that TPACK was preferred to be used as a metric for pre-service teachers' technology integration during a course design (e.g., Harvey & Caro, 2017). The profound analysis of these studies signifies that there is still a need to examine pre-service teachers' TPACK components during COVID-19 times to develop a research-based course design because very few up-to-date studies (e.g., Hall et al., 2020; Harvey & Caro, 2017) developed a course based on predefined TPACK components.

Mathematics, science, social studies, English language teaching, and other branches in the teacher education department have been the popular subject areas in which researchers aimed to explore pre-service teachers' perceptions, views, and experiences regarding TPACK (e.g., Kaur et al., 2021; Li, 2021, Shin et al., 2021). For instance, Harris and Hofer (2011) conducted a study with social studies in-service teachers, Kaur et al. (2021) and Li (2021) with English language teachers, Huang et al. (2021), and Jang and Tsai (2012) worked with elementary mathematics and science in-service teachers, and Shin et al. (2021) with science teachers. Different from these studies, as mentioned previously, we believe that pre-service teachers are a good fit for our study; hence, we selected pre-service mathematics teachers as a study group to investigate their perceived TPACK regarding the geometry subject area. Moreover, geometry is particularly chosen as it is not considered a hot topic as much as other branches in mathematics (e.g., numeracy, statistics), and studies in this field are progressing with more limited groups. In addition, we aimed to deal with all TPACK sub-dimensions in detail, which was not common in the reviewed literature hence we aimed to fill this gap in the field. Therefore, the research questions of the study are the following:

- What are the perceived TPACK levels of pre-service mathematics teachers?
- What are the perceived TPACK sublevels of these teachers? Namely:

  - pedagogy knowledge,
  - content knowledge,
  - pedagogical content knowledge,
  - technology knowledge,
  - technological pedagogy knowledge, and
  - technological content knowledge.
- Is there a gender difference in TPACK scores?

2. Methodology

2.1. Research Model

This study followed a survey research approach as a research methodology because we aimed to determine the specific characteristics of the study group (Fraenkel et al., 2014) so that we can design a course to help them improve in the knowledge domains where the participants think they need improvement (Herrington et al., 2007). To achieve this goal, participants in a descriptive survey were asked the same questions about TPACK as in Bulut and Işksal (2019)

2.2. Research Sample

The data were generated from 70 (29 males, 41 females) undergraduate students from the department of mathematics education. 41.4% of the participants were male (n = 29) and 58.6% of them (n = 41) were female. A purposeful sampling method was used to choose the sample according to the needs of the study (Fraenkel et al., 2014). We chose purposeful sampling as the longer aim of this research is to develop a course for those in the sample from the same university; hence, to meet their needs with the course, the researchers will continue accessing to the participants. The participants were the third or last year students of a university's Elementary Mathematics Education programme; they are also called pre-service mathematics teachers who will become middle school mathematics teachers in two years. The study sample corresponds to approximately 20% of the population (response rate).

Further descriptive information about the participants revealed that more than half of them attended a teaching practicum (n = 41, 58.6%) and half of them attended a teaching methods course (n = 35, 50%), while almost a quarter of the participants attended an advanced teaching methods course (n = 17, 24.35%). Finally, one-third of the participating pre-service teachers attended a school experience course (n = 23, 32.9%). This information is important for the current study as the effects of these variables (courses and experiences) were further analysed.

2.3. Data Collection Tools and Procedure

The Technology, Pedagogy and Content Knowledge (TPACK) Survey was prepared to evaluate the Technology, Pedagogy, and Content Knowledge of the students (pre-service teachers) studying at the Faculty of Education, Mathematics Education program as of the 2020-2021 academic year. The questions were from Bulut and Işıksal's (2019) survey. We chose Bulut and Işıksal's (2019) perceived geometry TPACK levels survey for various reasons. The first reason is that the survey was designed for the Turkish context, hence, no adaptation is needed, and from the introduction part to the open-ended items, it fits the goals of our study. Therefore, no amendments were needed. Secondly, the survey was designed in participants' language; hence, we did not need any translation or further work related to translation for the data collection. Finally, the survey was valid and reliable. For the survey, three expert opinions were taken for face validity, semi-structured and an exploratory factor analysis was conducted for the construct validity concerns. Then, the necessary changes on the items were made after evaluating opinions and analyses. Some items in the survey were unclear, so we amended the language for these items. These issues were related to the translation from English to Turkish, so the validity and reliability of the survey was checked. For reliability analysis of the survey, the Cronbach alpha coefficient was calculated for the whole instrument (0.96) and for each item (ranging from 0.83 to 0.92) by Bulut and Işıksal (2019). During the research process, the survey was found to be valid and reliable.

The survey consisted of two parts. In the first part, participants were asked to answer questions about their perceived TPACK (51 questions); in the second part, participants were asked to fill in demographic information (7 questions). Demographic questions were about their age, gender, grade level, courses they took on teaching methods, courses they took on technology, teaching experience, and future plans on technology integration. We asked for this information to see whether any of these affect their TPACK. The number of items (out of 51) which were related to each of the sub-dimensions was noted in order here, and described in detail and reported in the findings: pedagogy knowledge (items 1-8), technology knowledge (items 9-15), technological pedagogy knowledge (items 16-23), content knowledge (items 24-29), pedagogical content knowledge (items 30-37), technological content knowledge (items 38-43), and, techno-pedagogical content knowledge (items 44-51). The reliability value for our study was found as 0.97 for all items; and 0.88 for PK, 0.88 for TK, 0.92 for TPK, 0.82 for CK, 0.93 for PCK, 0.88 for TCK and 0.92 TPACK. There was a compelling match between the reliabilities of the source and the current study reflected the consistency of the results. A commonly accepted rule is that 0.6-0.7 indicates an acceptable level of reliability, and 0.8 or greater is a very good level; hence, these were all very good levels of reliability (Fraenkel et al., 2014).

2.4. Data Analysis

The TPACK framework was used to analyse of the survey (Mishra & Koehler, 2006). The framework had seven components, and there were 6 to 8 items for each component: pedagogy knowledge (8 items), technology knowledge (7 items), technological pedagogy knowledge (8 items), content knowledge (6 items), pedagogical content knowledge (8 items), technological content knowledge (6 items), and technological pedagogical content knowledge (8 items

As a reminder, all study participants were pre-service teachers even though some definitions include a teacher as the object. No other changes were needed to specify these definitions for our study; we used original definitions. We reported the survey data using these knowledge domains as the themes of our study.

Agreement percentages were calculated by adding the percentages of somewhat agree, agree and strongly agree; and disagreement percentages were calculated by adding the percentages of somewhat disagree, disagree and strongly disagree.

In addition to the Likert-type items, the findings present results of the open-ended items, hence it is important to describe how the data from such open-ended items were coded. The coding was done by rereading the answers a few times and then reporting the results in percentages. We measured the level of agreement between the raters to see the consistency to report reliability. The inter-rater consistency coefficient for coding the open-ended items was 92% which was high enough to report reliability. This was obtained by dividing the total number of ratings by the number of ratings in an agreement between two raters.

Finally, statistical analysis techniques such as t-tests and ANOVAs were used to determine the effects of mathematics teaching methods, educational technology, and internship courses on participants' TPACK scores. We looked at whether there was a gender difference (i.e, independent variable, IV) in terms of the sub-components of the TPACK as well as the overall TPACK scores (i.e., dependent variables, DV). Given that the literature suggested no gender difference in TPACK, we looked at whether there was any difference in the sub-components of TPACK.

2.5. Ethical

Upon granting permission by the Human Subjects Ethics Committee of the first author's university and subsequently by the Provincial Directorate of National Education, the study did not cause any concerns for the committee.

3. Findings

In this study, we analysed the participants' responses to the questions about TPACK (51 questions). This paper presents the findings of the survey with 70 pre-service mathematics teachers. The survey results are presented under seven components of the TPACK framework, namely pedagogy knowledge (section 4.1), technology knowledge (section 4.2), technological pedagogy knowledge (section 4.3), content knowledge (section 4.4), pedagogical content knowledge (section 4.5), technological content knowledge (Section 4.6), and technological pedagogical content knowledge (section 4.7). While reporting, we chose to use positive language where we reported the "agreement" results. That is, if 60% of participants partially agreed, agreed, or strongly agreed that "they were able to effectively integrate necessary technologies into their geometry instruction," we reported that percentage in our results rather than saying, 40% of participants think "they did not effectively integrate necessary technologies into their geometry instruction."

First of all, note that we found no gender difference neither in sub-components of TPACK (PK: p = .98, CK: p = .08, PCK: p = .63, TK: p = .32, TCK: p = .96, TPK: p = .32), nor in total TPACK scores (p = .83). Hence, this guided us to provide a comprehensive course suitable for both genders. Course structure is provided in Appendix A to share an insight for teacher educators who aim to design a TPACK course. Secondly, we looked at associations between grade level and TPACK total & subcomponent scores. We did not found any significant difference between the PSTs who were junior (M = 30.87, SD = 9.53), and senior grade level (M = 32.81, SD = 6.73) in terms of their TPACK scores; t(67) = -.99 , p > .05.

With an interest in the course and teaching experience, we further analysed the relationships between doing internship, teaching experience, and studying educational technology courses, and TPACK scores. Relying on independent sample t-test results,we found non-significant difference between the TPACK scores of PSTs who attended a teaching experience/ an internship course (M = 32.81, SD = 6.73), and those who did not attend any (M = 30.87, SD = 9.53); t(67) = .99, p > .05). Moreover, there was no difference in scores of those studying a mathematics teaching methods course (M = 32.97, SD = 6.60), and those who did not study (M = 30.62, SD = 9.70); t(64) = 1.20, p > .05). Lastly, we found that there was no difference between TPACK scores of PSTs who attended an educational technology course (M = 32.04, SD = 7.51), and those who did not attend (M = 31.13, SD = 10.05); t(37) = .39, p > .05) related to TPACK total. We also did the same analyses as 2x2 ANOVA, which gave similar results. For this, we first checked the assumptions: independent observation, normality and homogeneity of variance for, teaching experience (an IV), method course (an IV) and TPACK total scores (DV). All but the homogeneity of variance assumptions was not violated. F(3,66) = 4.10, p= .01 < .05. There was no interaction between teaching experience and method course variables which means that the effect of teaching experience does not depend on the levels of the method course. Then, a 2X2 ANOVA was conducted to evaluate the effects of teaching experience and enrolling in a method course on TPACK total scores. The analysis indicated no significant interaction between teaching experience and method course, F(1,66) = .01, p >.05, but significant main effect for teaching experience F(1,66) = 4.66, p = .04 < .05, $\eta2$ = .07 and method course, F(1,66) = .78, p > .05. As results indicated, PSTs who had teaching experience (M=.34.76, SD=7.93) tended to have different TPACK scores than those who did not have (M= 30.04, SD=8.27). Because there were only two variables, post-hoc analysis was not required, and the results showed that PSTs who had teaching experience had significantly higher TPACK scores.

The following provides the case in each of the components of TPACK. In each sub-components of the 51-itemed survey, we visualised our data with effective use of 100% stacked bar (Wilke, 2019) in the following subtitles of the findings.

3.1. Pedagogy Knowledge

According to Item 1, nearly four-fifths of the PSTs reflected their pedagogical knowledge that they are able to evaluate their students in-class performances (88.6%). Items 2, 3, 4, 5, 6, 7, and 8 indicated that a similar proportion of participants also adapt their teaching method to their students' needs (93.6%), organize their teaching method in accordance with their students' learning levels and individual differences (91.4%), select effective resources and activities to enrich their learning (94.3%), use different instructional methods (91.4%), manage their class effectively while lecturing (87.1%), and use different instructional methods.

Overall, the participants' perceived pedagogy knowledge levels were ~90%. These findings showed that most of the pre-service mathematics teachers (almost 80% on average) perceived themselves as proficient in terms of pedagogical knowledge (see Figure 1).

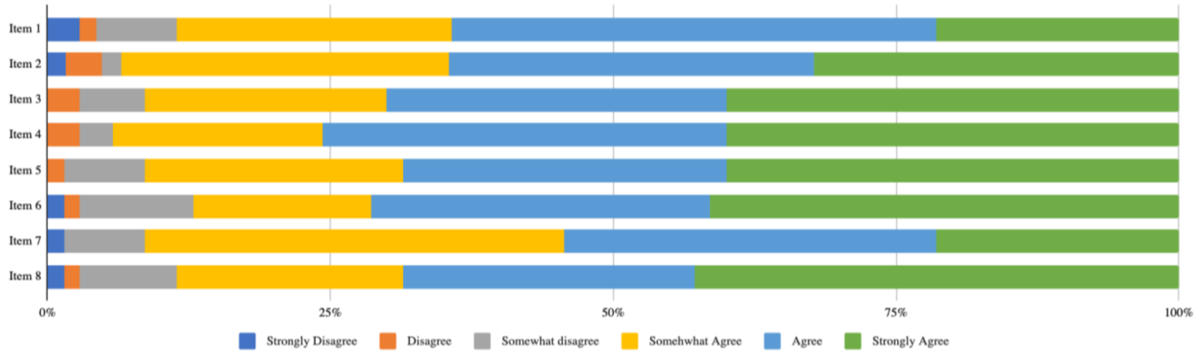

Figure 1. *Descriptive for the Pedagogical Knowledge Dimension of the Scale*

3.2. Technology Knowledge

Figure 2 shows the percentages of participants according to their answers to the perceived TPACK survey's technology knowledge dimension.

Items 11 and 12 showed that around four-fifths of the pre-service mathematics teachers believed that they could use basic computer software such as Windows and Office Tools (84.3%) and presentation tools such as projector and smartboard (82.9%) effectively. According to Item 9, more than two-thirds of the participants believed that they knew how to solve a technical problem while working on the computer (61.5%) although as item 10 indicated, only half of the PSTs reported that they knew the basic computer hardware parts including Video Card, Motherboard, Main Memory, and RAM, and their functions (52.8%). When it comes to software-related problems particularly (Item 15), nearly half of the PSTs thought that they could easily solve a software problem that they encountered on the computer (45.7%). Most participants thought they could easily learn to use newly encountered technologies, whether hardware or software (Item 13; 81.5%). Finally, as Item 14 showed, almost three-quarters of the participants believed they could easily find audio-visual technologies (animation, simulation, etc.) they were looking for via the Internet or by purchasing (74.3%).

Overall, the participants' perceived technology knowledge levels were ~60%. These items indicated that most of the participating pre-service mathematics teachers (69% on average) perceived themselves as proficient in technological knowledge although only half of them believed that they could solve software-related problems (see Figure 2).

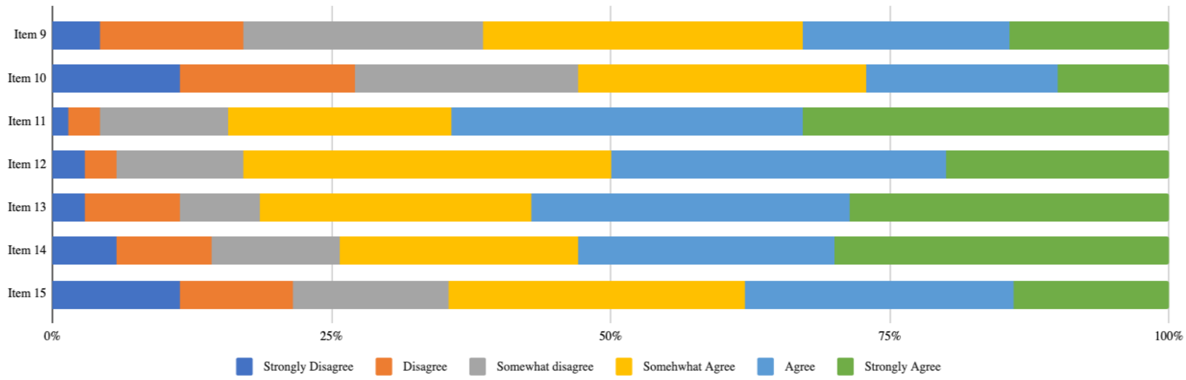

Figure 2. *Descriptive for Technology Knowledge Dimension of the Scale*

3.3. Technological Pedagogy Knowledge

Items 16, 19, and 21 respectively showed that around four-fifths of the PSTs believed that they could choose technologies that would make their teaching method effective (84.2%) and enrich the content of their course (82.8%) and that they could evaluate the practicality of a new technological tool in education (79.9%). Items 18, 20, 22, and 23 indicated that the participants believe that they can plan technology-based activities for their lessons (90%), that they could manage the classroom during technology-based activities (90%), that they could plan a lesson in a way that allows them to use technology effectively (88.6%), and that they have sufficient knowledge about teaching with technology (92.8%). Finally, and a little lower than the other percentages in this theme, around seven-tenths of the participants considered that they could determine the appropriate hardware or software technologies for the teaching method that they would use in their lessons (72.9%).

Overall, participants' perceived technology education knowledge level was ~70%. These results showed that almost all of the participating pre-service teachers (85.2% on average) considered themselves proficient in technological pedagogy knowledge (see Figure 3).

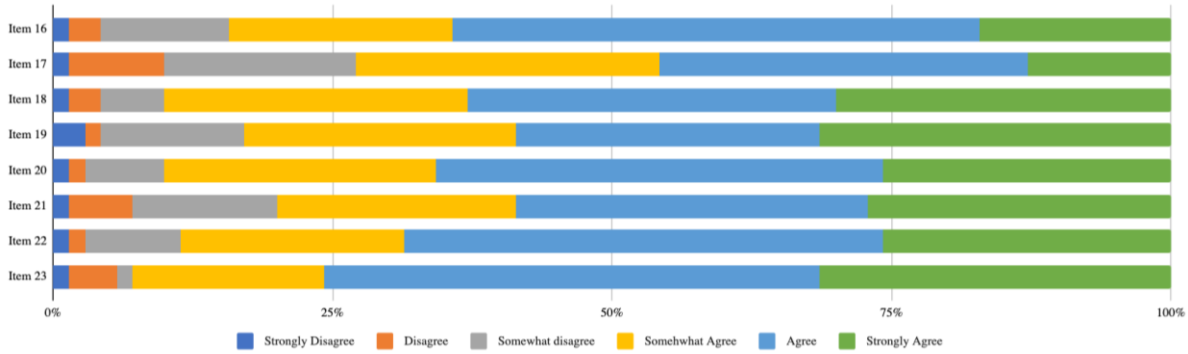

Figure 3. *Descriptive for Technological Pedagogical Knowledge Dimension of the Scale*

3.4. Content Knowledge

Almost nine-tenths of the PSTs reflected their content knowledge that in line with Items 24, 25, 26 and 27 respectively, they can answer questions about geometry topics (90%) easily, relate geometry topics to daily life (91.4%), associate mathematics with other learning areas and different disciplines including (e.g., Science and Technology, [National] Language, Social Studies, etc.) (92.9%), and do research to improve themselves (92.9%). According to Item 28, more than half of them reflected that they can explain mathematical concepts in geometry (e.g., line, point, angle) within the middle school mathematics curriculum (95.7%). Finally, Item 29 showed they can do proofs in the geometry topics within the middle school mathematics curriculum (81.4%).

Overall, the participants' total perceived content knowledge levels were ~85%. These findings showed that nearly all of the pre-service mathematics teachers (90.3%) perceived themselves as very proficient in terms of content knowledge in geometry (see Figure 4).

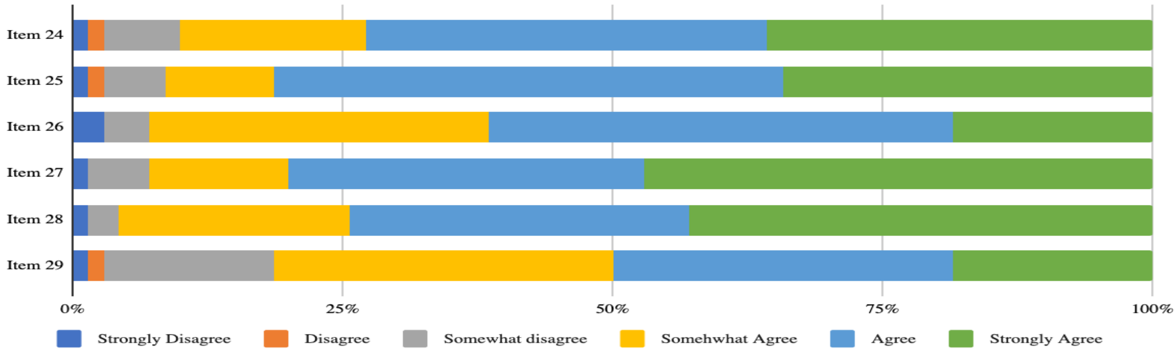

Figure 4. *Descriptive for Content Knowledge Dimension of the Scale*

3.5. Pedagogical Content Knowledge

Nearly 80% of PSTs showed that teachers had the pedagogical content knowledge to select teaching methods that would aid students in learning geometry topics (91.4%), to identify students' misconceptions about geometry (87.1%), to determine the reasons for students' misconceptions about geometry (84.3%), to do a lesson plan about geometry that would motivate their students to learn (92.9%), and to prepare activities that would help students apply what they had learned (92.9%). In addition, item 36 showed that most of them reflected that when they become a teacher, they can relate geometry to other mathematics subjects while teaching (92.9%). Moreover, item 32 and item 34 indicated that nearly all of the PSTs reflected that they can easily use different teaching methods (e.g., problem-solving) when teaching geometry (92.9%) and could measure their students' learning in geometry (97.1%).

Overall, total perceived pedagogical content knowledge levels of the participants were ~90%. These findings showed that most of the PSTs (91.1%) perceived themselves as very sufficient in terms of pedagogical content knowledge in geometry (see Figure 5).

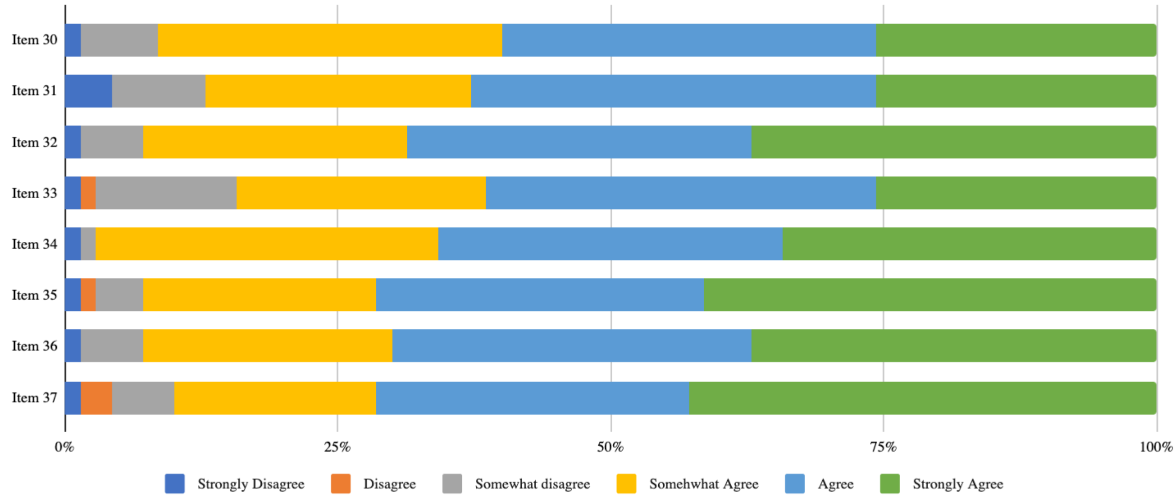

Figure 5. *Descriptives for Pedagogical Content Knowledge Dimension of the Scale*

3.6. Technological Content Knowledge

Items 38 and 39 indicated that around two-thirds of the PSTs believed that they knew what kind of technologies (computer, software, material, etc.) were used in geometry (71.4%) and which computer software (e.g., Geometer's Sketchpad, Logo, GeoGebra, C.A.R.) were available for geometry subjects (57.2%). From all the TCK items, these were the ones where agreement in responses was lowest. Items 40, 41, and 43 showed that 80% of participants knew which hardware technologies (projector, calculator, smartboard, etc.) could be used in geometry problems, 78% could effectively combine technology and their teaching method when teaching geometry, and 78% could assist other teachers in doing the same. Lastly, according to item 42, most PSTs tended to think that in the geometry course, they could choose technologies that would enrich their students' learning and teaching (90%). Item 42 was the only item in the TCK theme with more than 80% of the participants choosing the options in the "agreement" part.

Overall, the participants' perceived technological content knowledge levels were ~70%. The results indicated that most of the participating pre-service maths teachers (75.9% of them on average) perceived their technological content knowledge at the medium level (see Figure 6).

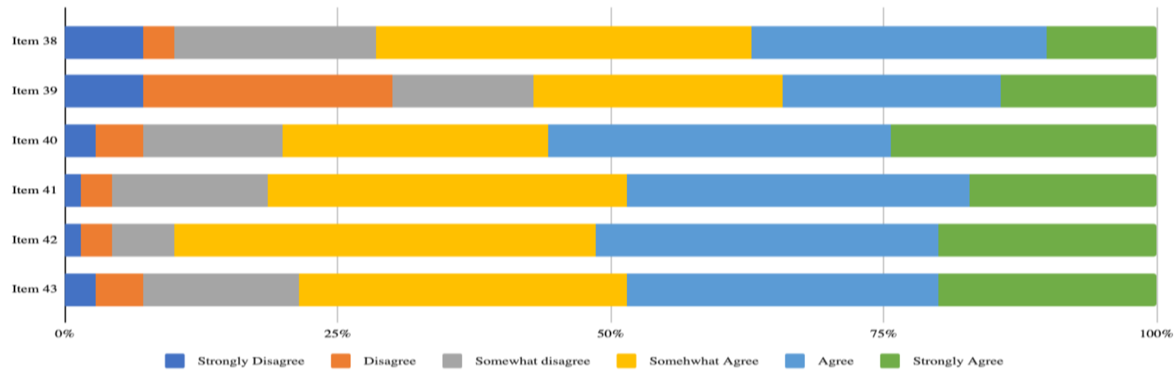

Figure 6. *Descriptive for Technological Content Knowledge Dimension of the Scale*

3.7. Techno-pedagogical Content Knowledge (TPACK)

In this context, TPACK implied pre-service mathematics teachers' awareness of the connections between technology, pedagogy, and the content of geometry. According to items 44, 45, 47, and 51, more than three quarters of the pre-service maths teachers believed that they could effectively explain geometry topics using technology and various teaching methods (84.3%), visual and auditory technologies (animations, simulations, etc.) during geometry lessons with ease (84.3%), can plan their geometry lesson using an effective combination of technology and their chosen teaching method (80%), and can effectively assess student learning levels using dynamic geometry and math software during geometry lessons (75.7%). Items 46 and 50 indicated that more than two-thirds of the PSTs thought they could easily solve a problem (hardware or software) that students encounter when using technology in geometry teaching (64.2%) and using dynamic geometry and mathematics software to teach geometry subjects (67.1%). For item 48, half of the PSTs stated that they knew how to use dynamic geometry and mathematics software (Geometer's Sketchpad, GeoGebra, Cabri, etc.) effectively (50%). Finally, according to item 49, two-fifths of the PSTs declared that they knew how to solve a problem using dynamic geometry and maths software (40.1%).

Overall, participants' perceived TPACK level was ~60%. These findings indicated that most of the participating pre-service maths teachers (68.2% of them on average) perceived their technological pedagogical content knowledge at the medium level (see Figure 7).

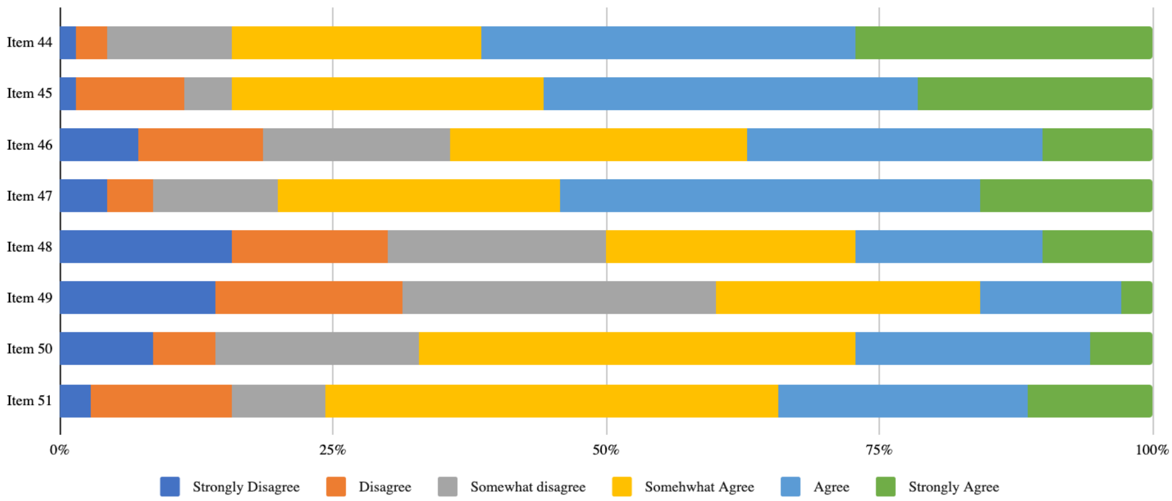

Figure 6. *Descriptive for General TPACK Dimension of the Scale*

In general, the findings showed that pre-service teachers were aware of the necessity of the technological pedagogical content knowledge (TPACK), and the effective integration of its components. The participating pre-service teachers indicated they preferred to frequently use technology in their future teaching experiences (n= 44, 62.9%). In comparison, 27.1% of them prefer to use technology "often", 7.1% of them prefer to "always" use technology, and 2.9% of them prefer to use technology "rarely" (see Figure 8).

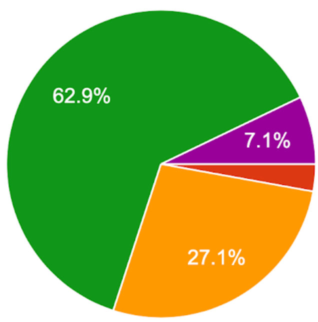

Figure 8. *Technology Use*

4. Conclusion and Discussion

To conclude, we can infer from the findings that pre-service teachers may not have the full teaching experience in practicum schools. Despite the practicum courses, most participants still feel inexperienced in teaching. This sets the environment for our second cycle, where we plan to design a course to provide such an experience.

In general, although pre-service teachers has enjoyed the idea of integrating technology into their lessons (Corkett & Benevides, 2015), we found that pre-service teachers need extra support for all of the TPACK sub-components, but particularly for those including technology component: technology knowledge, technological content knowledge, technological pedagogy knowledge and TPACK itself. This finding mostly confirmed the wider literature which helped us develop a course students and those who need similar competencies in other contexts (Agyei & Voogt, 2015; Koh & Chai, 2011; Tondeur et al., 2019). It gave us hope for further adaptations for use in classes of wider Europe and America when we consider the results of the studies conducted in these contexts (Bayaga et al., 2021; Niess, 2008).

Contrary to the literature, one of the intesting results of this study was that the TPACK scores of the PSTs who enrolled in a method course, technology course, and internship course were similar to those who did not. However, Mouza et al. (2017) explained to us in detail the effect of the computer technologies course on TPACK scores. In addition, Huang and colleagues (2021) expressed the relation between attending technology courses with TPACK-related skills. As mathematics teachers who have graduated from a faculty of education, we know that taking regular courses allows us to keep the information up to date. We actively participate in the lesson, listen to the instructor, and do homework. However, if daily life examples related to the course content are not shared (Çam & Erdamar-Koç, 2021), the knowledge cannot be kept up-to-date.

Today, the fact that the faculties of education is subject-based and still mostly traditional (İlter, 2014) does not work for current needs of PSTs. The education provided by the faculties in Turkey is mostly theory-based (Çakıroğlu & Çakıroğlu, 2003). The development of TPACK skills and PSTs' own perception regarding TPACK can be attributed to the integration of technology in the faculty courses (Yıldız Durak, 2021) and their enrichment with activities and daily life examples (Çam, 2018; Mourlam et al., 2021). The courses enrolled by the PSTs may not have included this kind of instructional design sufficiently. In addition, it is a salient finding that the courses offered by instructors who have low content knowledge or pedagogic knowledge and teach only technology-based do not increase the TPACK score of the students (Mishra & Kochler, 2006). Hence, in this study, perhaps the courses opened in various faculties in Turkey are still being prepared traditionally and theoretically, and may even have been offered by instructors who did not train themselves in TPACK.

Moreover, confirming the literature, we did not find any gender difference in TPACK (Al-Abdullatif, 2019; Altun, 2019; Koh & Chai, 2011; Schmid et al., 2021). We further analysed gender difference for sub-components of TPACK in case there was any difference in subcomponents. The findings for subcomponents were no different than those for total TPACK, with no gender difference. Although adding to the literature about the analysis of the sub-components (Adalar, 2021), we could conclude that as in most cases, there was also no gender difference in case of TPACK subcomponents in mathematics.

Finally, all of the participating pre-service teachers reflected on their teaching practices, which is argued to be a factor affecting students' mathematics performance. Research has shown that students' errors in mathematics might occur because students have difficulties in understanding teachers' instruction methods (Confrey, 1990; Saralar-Aras, 2022). Our study confirmed previous studies (Çelik, 2013; Dönmez et al., 2021; Gür & Seyhan, 2006), and found that pre-service teachers value teaching practices, including how they received and delivered instruction.

5. Recommendations

First of all, a purposive sampling method was used to choose the sample according to the needs of the study. Using a non-random sampling method limited the generalizability of research (Fraenkel et al., 2014). However, the longer-term goal of this study was to design a course that would meet the needs of the participants in each context. Still, the study with the same survey might be repeated in other contexts in Turkey and wider Europe, and with a greater number of participants to add to these results. Secondly, the participants were selected from the third and fourth (senior) pre-service mathematics teachers of a university's elementary mathematics education program. The study can be replicated with the first and second-year pre-service teachers to see whether there is any difference in the TPACK at the beginning and end years of their teacher education. Lastly, the content knowledge in this study was limited to geometry, as we focussed on this and used Bulut and Işıksal's (2019) survey, specifically designed to assess perceived TPACK in geometry. As a result, it was a limitation, and a replication of this study with different contents via surveys tailored to these contents could be proposed.

5.1. Implications

This study has various implications for pre-service mathematics teachers, teachers, teacher educators, and researchers. First, we observed the TPACK knowledge domains pre-service teachers thought they had difficulty. Based on the analysis of the results, we will design a teaching experience course for them to improve their knowledge in the missing domains. We plan to ask about their experiences of teaching, in addition to assessing their TPACK. Such research in real teaching settings might help not only pre-service teachers to gain necessary practice and knowledge, and gain self-awareness about their practices but also allow students to take lessons from teachers who have necessary techno-pedagogical knowledge and related skills (TPACK) as Mishra and Koehler (2006) suggested. Reflecting on their knowledge and thinking about their future practices might also be argued to be a factor which might help pre-service teachers to improve their future teaching practices and prevent their students' possible errors and misconceptions in geometry (Confrey, 1990; Leijen et al., 2020; Lim, 2011; Pusey, 2003).

Moreover, the related literature shows that pre-service teachers' TPACK increases with the teaching experience (Balgalmış, 2013; Jang & Tsai, 2012; Saralar et al., 2018). The results of this study showed that pre-service teachers do not have the necessary teaching experience in real classrooms before they start their in-service teaching. When the PSTs are provided with such experience through a designed course, they might have increased technological, and pedagogical content knowledge.

We also believe that technology teachers can participate in mathematics classes at middle schools for just one hour a week with collaborative work and teach mathematics teachers and their students basic technological tools. During the guidance course hour in the weekly program, mathematics teachers can research, learn and practice geometry focus software packages, focusing on their professional development (while their students are doing assignments at the same time). Then, they can use it in the in-class teaching in the following weeks. Technology teachers can be invited as guest speakers during elective mathematics hours. They can discover novice geometry-focused hardware and software tools in collaborative work as a workshop study. Thus, teachers' and pre-service teachers' technology literacy, awareness, and self-confidence in TPACK knowledge and frequency of usage of those skills can be increased in line with Voogt and McKenney's (2007) findings. They can easily solve a problem (hardware or software) students encounter when using technology in geometry teaching.

Finally, teacher educators could use the same survey and design similar settings for pre-service teachers at their universities, and then expand on this research with larger samples in geometry education, other branches of mathematics such as algebra and calculus, and other fields of education such as science education and language education.

Declaration: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

6. References

Abunda, N. D. (2021). Development of a TPACK-based teacher's module in plane coordinate geometry. *International Journal for Technology in Mathematics Education, 28*(4), 225-233. https://l24.im/t9VHXBw

Açıkgül, K., & Aslaner, R. (2019). Investigation relations between the technological pedagogical content knowledge efficacy levels and self-efficacy perception levels of pre-service mathematics teachers. *Çukurova University Faculty of Education Journal, 48*(1), 1-31. https://10.14812/cufej.409949

Açıkgül, K., & Aslaner, R. (2020). Effects of Geogebra supported micro-teaching applications and TPACK game practices on the TPACK levels of prospective teachers. *Education and Information Technologies, 25*(3), 2023-2047. https://doi.org/10.1007/s10639-019-10044-y

Adalar, H. (2021). Social studies teacher candidates' self-efficacy beliefs for technological pedagogical content knowledge (TPACK). *International Journal of Education and Literacy Studies, 9*(3), 169-183.

Agyei, D. D., & Voogt, J. M. (2015). Pre-service teachers' TPACK competencies for spreadsheet integration: insights from a mathematics specific instructional technology course. *Technology, Pedagogy and Education.* https://doi.org/10.1080/1475939X.2015.1096822

Al-Abdullatif, A. M. (2019). Auditing the TPACK confidence of pre-service teachers: The case of Saudi Arabia. *Education and Information Technologies, 24*(6), 3393-3413.

Alizadeh-Jamal, M., Shahvarani, A., Iranmanesh, A., & Tehranian, A. (2018). A study on the changes on teachers' knowledge and beliefs after a workshop based on maths education software, by relying on fuzzy method. PNA. *Revista de Investigación en Didáctica de la Matemática, 13*(1), 19-40. https://doi.org/10.30827/pna.v13i1.6593

Altun, D. (2019). Investigating pre-service early childhood education teachers' technological pedagogical content knowledge (TPACK) Competencies regarding digital literacy skills and their technology attitudes and usage. *Journal of Education and Learning, 8*(1), 249-263.

Balgalmış, E. (2013). *An investigation of pre-service elementary mathematics teachers' techno-pedagogical content knowledge within the context of their teaching practices* [Doctoral dissertation]. Middle East Technical University.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A meta-analysis of the relation between mathematics anxiety and mathematics achievement. *Psychological Bulletin, 147*(2), 134–168. https://doi.org/10.1037/bul0000307

Bayaga, A., Bosse, M. J., Sevier, J., Fountain, C., Williams, D., Bosire, S., & Blignaut, S. (2021). University faculty opinions of preservice teachers' technological readiness. *Canadian Journal of Science, Mathematics and Technology Education, 21*(1), 44–64. https://doi.org/10.1007/s42330-021-00138-6

Birgili, B., Kiziltepe, Z., & Seggie, F. N. (2016). Teaching method preferences of teachers: The Cooperative teaching method. *World Studies in Education, 17*(2), 37–52. https://doi.org/10.7459/wse/17.2.04

Bottge, B. A. (1999). Effects of contextualized math instruction on problem-solving of average and below-average achieving students. *The Journal of Special Education, 33*(2), 81–92. https://doi.org/10.1177/002246699903300202

Bulut, A., & Işıksal, M. (2019). Perceptions of pre-service elementary mathematics teachers on their technological pedagogical content knowledge (TPACK) regarding geometry. *Journal of Computers in Mathematics and Science Teaching, 38*(2), 153–176. https://www.learntechlib.org/primary/p/173761/

Çakıroğlu, E., & Çakıroğlu, J. (2003). Reflections on teacher education in [Nationality]. *European Journal of Teacher Education, 26*(2), 253-264. https://doi.org/10.1080/0261976032000088774

Çam, Ş. S. (2018). *Öğretim elemanlarının teknolojik pedagojik alan bilgilerinin geliştirilmesi için bir mesleki gelişim program önerisi* [Doktora tezi]. Gazi Üniversitesi, Ankara.

Çam, Ş. S., & Erdamar Koç, G. (2021). Technological pedagogical content knowledge practices in higher education: First impressions of preservice teachers. *Technology, Knowledge and Learning, 26*(1), 123-153. https://doi.org/10.1007/s10758-019-09430-9

Cancino, M., Durán, M., & Solorza, C. (2020). What learning can do to teaching: assessing the impact of apprenticeship of observation on pre-service teachers' beliefs. *English Teaching & Learning, 44*, 297–312. https://doi.org/10.1007/s42321-019-00044-z

Çelik, S. (2013). *İlköğretim matematik derslerinde kullanılan alternatif öğretim yöntemlerinin akademik başarıya etkisi: Bir meta analiz çalışması* (Master's thesis]. Osmangazi Üniversitesi, Eskişehir.

Choi, B., & Young, M. F. (2021). TPACK-L: Teachers' pedagogical design thinking for the wise integration of technology. *Technology, Pedagogy and Education, 30*(2), 217–234. https://doi.org/10.1080/1475939X.2021.1906312

Clements, D. H. (2003). Teaching and learning geometry. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *Research companion to principles and standards for school mathematics* (pp. 151-178). Reston, VA: NCTM.

Cobb, P., Confrey, J., diSessa, A. A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13. https://10.3102/0013189X032001009

Confrey, J. (1990). A review of the research on student conceptions in mathematics, science, and programming. *Review of Research in Education, 16*(1), 3–56. https://doi.org/10.2307/1167350

Corkett, J., & Benevides, T. (2015). Pre-service teachers' perceptions of technology and multiliteracy within the inclusive classroom. *International Journal of Psychology and Educational Studies, 2*(2), 35–46. https://doi.org/10.17220/ijpes.2015.02.004

Darling-Hammond, L. (2006). *Powerful teacher education: Lessons from exemplary programs*. Jossey-Bass: A Wiley Imprint.

Dönmez, B., Dönmez, K. H., Kolukısa, Ş., & Yılmaz, Ş. (2021). İlkokul matematik dersinde oyunla öğretim yöntemi kullanılmasının tutum ve başarıya etkisi. *Uluslararası Bozok Spor Bilimleri Dergisi, 2*(2), 58-70.

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology, 7*. https://doi.org/10.3389/fpsyg.2016.00508

Ferrarello, D., Mammana, M. F., Pennisi, M., & Taranto, E. (2017). Teaching intriguing geometric loci with DGS. In Aldon, G., Hitt, F., Bazzini, L., Gellert, U. (Eds) *Mathematics and technology*: *advances in mathematics education*. Springer. https://doi.org/10.1007/978-3-319-51380-5_26

Fraenkel, J. R., & Wallen, N. E. (2014). *How to design and evaluate research in education*. McGraw-Hill.

Fraenkel, J., Wallen, N., & Hyun, H. (2014). Survey research. In *How to design and evaluate research in education* (pp. 390-421). McGraw-Hill Education Press.

Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Perceived learning environment and students' emotional experiences: A multilevel analysis of mathematics classrooms. *Learning and Instruction, 17*(5), 478-493. https://doi.org/10.1016/j.learninstruc.2007.09.001

Galanti, T. M., Baker, C. K., Morrow-Leong, K., & Kraft, T. (2020). Enriching TPACK in mathematics education: using digital interactive notebooks in synchronous online learning environments. *Interactive Technology and Smart Education, 18*(3), 345–361. https://doi.org/10.1108/ITSE-08-2020-0175

Gu, P., Lee, W. M., Roubidoux, M. A., Yuan, J., Wang, X., & Carson, P. L. (2016). Automated 3D ultrasound image segmentation to aid breast cancer image interpretation. *Ultrasonics, 65*(1), 51–58. https://doi.org/10.1016/j.ultras.2015.10.023

Gür, H., &, Seyhan, G. (2006). İlköğretim 7. sınıf matematik öğretiminde aktif öğrenmenin öğrenci başarısı üzerine etkisi. *Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi 8*(1), 17-27.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, latino school. *Journal for Research in Mathematics Education, 34*(1), 37–73. https://doi.org/10.2307/30034699

Haji, K., Royse, A., Green, C., Botha, J., Canty, D., & Royse, C. (2016). Interpreting diaphragmatic movement with bedside imaging, review article. *Journal of Critical Care, 34*(1), 56–65. https://doi.org/10.1016/j.jcrc.2016.03.006

Hall, J.A., Lei, J., & Wang, Q. (2020). The first principles of instruction: an examination of their impact on pre-service teachers' TPACK. *Education Tech Research Dev, 68,* 3115–3142. https://doi.org/10.1007/s11423-020-09866-2

Harris, J. B. (2016). In-service teachers' TPACK development: Trends, models, and trajectories. In *Handbook of technological pedagogical content knowledge (TPACK) for educators* (pp. 201-216). Routledge Press.

Harris, J., & Hofer, M. (2009, March). *Instructional planning activity types as vehicles for curriculum-based TPACK development.* In Society for Information Technology & Teacher Education International Conference (pp. 4087–4095). Association for the Advancement of Computing in Education (AACE).

Harris, J.B., & Hofer, M.J. (2011). Technological pedagogical content knowledge (TPACK) in action: A descriptive study of secondary teachers' curriculum-based, technology-related instructional planning. *Journal of Research on Technology in Education, 43*(3), 211–229.

Harvey, D.M., & Caro, R. (2017). Building TPACK in preservice teachers through explicit course design. *TechTrends, 61*, 106–114. https://doi.org/10.1007/s11528-016-0120-x

Herrington, J., McKenney, S., Reeves, T., & Oliver, R. (2007). Design-based research and doctoral students: Guidelines for preparing a dissertation proposal. *Edith Cowan University Research Online*, 1–10. https://ro.uow.edu.au/edupapers/627

Hew, K. F., & Brush, T. (2007). Integrating technology into K-12 teaching and learning: Current knowledge gaps and recommendations for future research. *Educational Technology Research and Development*, *55*(3), 223–252. https://doi.org/10.1007/s11423-006-9022-5

Huang, L., & Lajoie, S. P. (2021). Process analysis of teachers' self-regulated learning patterns in technological pedagogical content knowledge development. *Computers & Education, 166*, 104169. https://doi.org/10.1016/j.compedu.2021.104169

Huang, X., Lai, M. Y., & Huang, R. (2021). Teachers' learning through an online lesson study: An analysis from the expansive learning perspective. *International Journal for Lesson & Learning Studies, 10*(2), 202-216. https://doi.org/10.1108/ijlls-09-2020-0076

İlter, İ. (2014). Opions of teacher candidates on the methods and techniques used by their lecturers. *Journal of International Social Research*, 7(35).

Jang, B. S. (2021). The analysis of preservice teachers' perception of TPACK. *Journal of Practical Engineering Education*, *13*(2), 293–300. https://doi.org/10.14702/JPEE.2021.293

Jang, S.-J., & Tsai, M.-F. (2012). Exploring the TPACK of Taiwanese elementary mathematics and science teachers with respect to the use of interactive whiteboards. *Computers & Education 59*(2), 327–338. https://doi.org/10.1016/j.compedu.2012.02.003

Kaur, A. P. R., Singh, H., & August, A. L. (2021). Exploring the factor structure of the constructs of technological, pedagogical, and content knowledge (TPACK): An exploratory factor analysis based on the perceptions of TESOL pre-Service teachers. *Research Journal of Education*, *7*(2), 103–115. https://ideas.repec.org/a/arp/rjearp/2021p103-115.html

Kaya, V. H., & Elster, D. (2019). Environmental science, technology, engineering, and mathematics pedagogical content knowledge: Teacher's professional development as environmental science, technology, engineering, and mathematics literate individuals in the light of experts' opinions. *Science Education International*, *30*(1), 11–20. http://www.icaseonline.net/journal/index.php/sei/article/view/101

Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education, 9*(1), 60–70. https://www.learntechlib.org/primary/p/29544/

Koh, J. H. L., & Chai, C. S. (2011, December). *Modelling pre-service teachers' technological pedagogical content knowledge (TPACK) perceptions: The influence of demographic factors and TPACK constructs*. Paper presented at the Australasian Society for Computers in Learning in Tertiary Education (ASCILITE) Conference, Hobart, Tasmania, Australia. https://repository.nie.edu.sg/bitstream/10497/14126/3/Ascilite-2011-735_a.pdf

Koh, J. H. L., & Chai, C. S. (2014). Teacher clusters and their perceptions of technological pedagogical content knowledge (TPACK) development through ICT lesson design. *Computers & Education*, *70*, 222–232. https://doi.org/10.1016/j.compedu.2013.08.017

Larkin, K., & Jorgensen, R. (2016). 'I hate maths: Why do we need to do maths?' Using iPad video diaries to investigate attitudes and emotions towards mathematics in year 3 and year 6 students. International *Journal of Science and Mathematics Education, 14*(5), 925–944. https://doi.org/10.1007/s10763-015-9621-x

Leijen, Ä., Pedaste, M., & Lepp, L. (2020). Teacher agency following the ecological model: How it is achieved and how it could be strengthened by different types of reflection. *British Journal of Educational Studies, 68*(3), 295–310. https://doi.org/10.1080/00071005.2019.1672855

Li, H. (2021). *The designing of CSL teacher empowering training model of robot-assisted language learning based on the TPACK framework.* Proceedings of 2021 International Conference on Advanced Learning Technologies [ICALT], (pp. 276–278). Tartu, Estonia. https://doi.org/10.1109/ICALT52272.2021.00089

Lim, S. K. (2011). Applying the Van Hiele theory to the teaching of secondary school geometry. *Teaching and Learning, 13*(1), 32–40. http://hdl.handle.net/10497/1544

Lin, T.-C., Tsai, C.-C., Chai, C.S., & Lee, M.-H. (2013). Identifying science teachers' perceptions of technological pedagogical and content knowledge (TPACK). *Journal of Science Education and Technology*, *22*(3), 325-336. https://doi.org/10.1007/s10956-012-9396-6

Lindquist, M. M., & Clements, D. H. (2001). Geometry must be vital. *Teaching Children Mathematics, 7*(7), 409–409. https://l24.im/aelWAHu

Lochhead, J, & Whimbey, A. (1987). Teaching analytical reasoning through thinking aloud pair problem solving. *New Directions for Teaching and Learning, 30*(1), 73–92. https://doi.org/10.1002/tl.37219873007

Lu, X., Clements-Croome, D., & Viljanen, M. (2012). Fractal geometry and architecture design: case study review. *Chaotic Modeling and Simulation (CMSIM),* 2(1), 311–322. https://www.takmaghale.com/uploads/product/pjfxwm_149268230096516.pdf

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017–1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x

Mourlam, D., Chesnut, S. R., & Bleecker, H. (2021). Exploring preservice teachers self-reported and enacted TPACK after participating in a learning activity types short course. *Australasian Journal of Educational Technology*, *37*(3), 152–169. https://doi.org/10.14742/ajet.6310

Mouza, C., Yang, H., Pan, Y. C., Ozden, S. Y., & Pollock, L. (2017). Resetting educational technology coursework for pre-service teachers: A computational thinking approach to the development of technological pedagogical content knowledge (TPACK). *Australasian Journal of Educational Technology, 33*(3). https://doi.org/10.14742/ajet.3521

Niess, M. (2008). *Mathematics teachers developing technology, pedagogy and content knowledge (TPACK).* In K. McFerrin, R. Weber, R. Carlsen & D. Willis (Eds.), Proceedings of SITE 2008-Society for Information Technology & Teacher Education International Conference, (pp. 5297-5304). Las Vegas, Nevada, USA: Association for the Advancement of Computing in Education (AACE). https://www.learntechlib.org/primary/p/28121/

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education, 21*, 509–523. http://one2oneheights.pbworks.com/f/Niess_M.pdf

Niess, M. L. (2011). Investigating TPACK: Knowledge growth in teaching with technology. *Journal of Educational Computing Research, 44*(3), 299–317. https://doi.org/10.2190/EC.44.3.c

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education, 9*(1), 4–24. https://psycnet.apa.org/record/2009-19577-001

Pusey, E. L. (2003). *The Van Hiele Model of reasoning in geometry: A literature review* [Master's thesis]. North Carolina State University.

Sahlberg, P. (2012). The most wanted: Teachers and teacher education in Finland. Darling-Hammond, L. & Lieberman, A. (Eds.). *Teacher education around the world: Changing policies and practices* (pp. 1–21). Routledge: Taylor and Francis Press.

Santos, J. M., & Castro, R. D. R. (2021). Technological pedagogical content knowledge (TPACK) in action: Application of learning in the classroom by pre-service teachers (PST). *Social Sciences & Humanities Open, 3*(1), 100110. https://doi.org/10.1016/j.ssaho.2021.100110

Saralar, İ., Işıksal-Bostan, M., & Akyüz, D. (2018). The evaluation of a pre-service mathematics teacher's TPACK: A case of 3D shapes with GeoGebra. *The International Journal for Technology in Mathematics Education, 25*(2), 3–21. https://doi.org/10.1564/tme_v25.2.01

Saralar-Aras, İ. (2022). RETA model for mathematics teaching: From the United Kingdom to Turkey. *In Global Perspectives and Practices for Reform-based Mathematics Teaching* (pp. 42-78). IGI Global.

Saralar-Aras, İ., & Esen, B. (2021). Geometri eğitiminde STEM çalışmaları. İ. Saralar-Aras (Ed.), *Okul öncesinden ortaöğretime farklı disiplinlerde STEM eğitimi uygulamaları* (s. 207-232). Millî Eğitim Bakanlığı.

Schmid, M., Brianza, E., & Petko, D. (2021). Self-reported technological pedagogical content knowledge (TPACK) of pre-service teachers in relation to digital technology use in lesson plans. *Computers in Human Behavior, 115*, 106586. https://doi.org/10.1016/j.chb.2020.106586

Shin, N., Bowers, J., & Krajcik, J. (2021). Promoting computational thinking through project-based learning. *Disciplinary and Interdisciplinary Science Education Research 3*(7). https://doi.org/10.1186/s43031-021-00033-y

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–11. https://doi.org/10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching foundations of the new reform. *Harvard Educational Review, 57*, 1–22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Skemp, R. R. (1987). *The psychology of learning mathematics.* Lawrence Erlbaum.

Smith, R. C., Kim, S., & McIntyre, L. (2016). Relationships between prospective middle grades mathematics teachers' beliefs and TPACK. *Canadian Journal of Science, Mathematics and Technology Education, 16*(4), 359-373. https://doi.org/10.1080/14926156.2016.1189624

Tahta, D. (1980). About geometry. *For the learning of mathematics, 1*(1), 2–9. https://www.jstor.org/stable/40247694

The United Nations (2015). *Transforming our world: The 2030 agenda for sustainable development.* https://sdgs.un.org/2030agenda

Thohir, M. A., Jumadi, J., & Warsono, W. (2020). Technological pedagogical content knowledge (TPACK) of pre-service science teachers: A Delphi study. *Journal of Research on Technology in Education*, https://doi.org/10.1080/15391523.2020.1814908

Tondeur, J., Scherer, R., Siddiq, F., & Baran, E. (2019). Strategies to prepare preservice teachers for Technological Pedagogical Content Knowledge (TPACK): A mixed-method study. *Educational Technology Research & Development.* https://doi.org/10.1007/s11423-019-09692-1

Tutak, F. A., & Adams, T. L. (2015). A study of geometry content knowledge of elementary preservice teachers. *International Electronic Journal of Elementary Education, 7*(3), 301–318.

Van Petegem, P., Donche, V., & Vanhoof, J. (2005). Relating pre-service teachers' approaches to learning and preferences for constructivist learning environments. *Learning Environments Research, 8*(3), 309–332. https://doi.org/10.1007/s10984-005-1564-7

Voogt, J., & McKenney, S. (2017). TPACK in teacher education: Are we preparing teachers to use technology for early literacy? *Technology, Pedagogy and Education, 26*(1), 69–83. https://doi.org/10.1080/1475939X.2016.1174730

Wang, A.Y. (2022). Understanding levels of technology integration: A TPACK scale for EFL teachers to promote 21st-century learning. *Educ Inf Technol, Online.* https://doi.org/10.1007/s10639-022-11033-4

Wilke, C. O. (2019). *Fundamentals of data visualization.* O'Reilly Press.

Wright, S. P., Horn, S. P., & Sanders, W. L. (1997). Teacher and classroom context effects on student achievement: Implications for teacher evaluation. *Journal of Personnel Evaluation in Education, 11*(1), 57–67. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.581.9316&rep=rep1&type=pdf

Yildiz Durak, H. (2021). Modeling of relations between K-12 teachers' TPACK levels and their technology integration self-efficacy, technology literacy levels, attitudes toward technology and usage objectives of social networks. *Interactive Learning Environments, 29*(7), 1136-1162.

Young, J. R., Young, J., Hamilton, C., & Pratt, S. S. (2019). Evaluating the effects of professional development on urban mathematics teachers TPACK using confidence intervals. *Redimat, 8*(3), 312–338. https://doi.org/10.17583/redimat.2019.3065

Za'ba, N., Ismail, Z., Abdullah, A. (2020). Preparing student teachers to teach mathematics with GeoGebra. *Universal Journal of Educational Research, 8*(5A), 29–33. https://doi.org/10.13189/ujer.2020.081904

Zambak, V. S., & Tyminski, A. M. (2020). Examining mathematical technological knowledge of pre-service middle grades teachers with a geometer's sketchpad in a geometry course. *International Journal of Mathematical Education in Science and Technology, 51*(2), 183–207. https://doi.org/10.1080/0020739X.2019.1650302

7. Appendices

Appendix A.

Table 2. *Course Design Outline for Development of TPACK in Geometry*

Weeks	Weekly Hour	Learning Themes	Issues to Discuss
1-3	3 (x3 weeks)	Technology knowledge	Use of standard sets of software tools such as word processors, spreadsheets, as well as browsers, and e-mail
4-7	3 (x4 weeks)	Technological pedagogy knowledge	Existence, components, and capabilities of various technologies (e.g., as assessment tools such as Kahoot and Socrative) as they are used in teaching and learning settings with particular teaching methods, e.g., inquiry learning, project-based learning and problem-based learning
8-10	3 (x3 weeks)	Technological content knowledge	Work on discovering how technology and content are reciprocally related and they support each other through tools that are specifically designed for geometry such as GeoGebra and Cabri
11-14	3 (x4 weeks)	Technological pedagogical content knowledge	Micro-teaching on how to use a chosen technology (e.g., GeoGebra) to teach a particular geometry topic from the middle school maths programme of the in Turkey (e.g., polygons) through a chosen teaching method (e.g., discovery learning)