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Pre-Service Mathematics Teachers' Understanding of Quadrilateral Definitions and Hierarchical Relationships: A Van Hiele Perspective*

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ABSTRACT

This study aims to examine pre-service elementary mathematics teachers' knowledge of quadrilaterals and evaluate their understanding within the framework of Van Hiele's Geometric Thinking Levels. In particular, the study investigates how pre-service teachers define quadrilaterals and establish relationships between different types of quadrilaterals and analyzes whether these processes differ across grade levels. The topic of quadrilaterals holds a crucial role in geometry instruction, as it requires not only familiarity with geometric properties but also a deep comprehension of hierarchical relationships among quadrilaterals. This highlights the direct impact of pre-service teachers' conceptual knowledge on students' geometry learning processes. The research was conducted using the cross-sectional approach within the general survey model. The study group consisted of 132 pre-service teachers enrolled in the 1st, 2nd, and 3rd years of the elementary mathematics teacher education program. Data were collected using an assessment form consisting of 12 multiple-choice questions, developed based on a literature review and expert opinions. This assessment tool was designed to measure pre-service teachers' ability to define quadrilaterals, establish relationships between them, and organize these relationships within a hierarchical structure. The analysis revealed that pre-service teachers primarily recognize quadrilaterals through prototypical representations, yet they struggle to define special cases and establish hierarchical relationships between quadrilaterals. Notably, the hierarchical relationship between deltoids and rhombuses was frequently overlooked. Although some conceptual gains were observed with increasing grade levels, no significant improvement was detected in terms of relating different types of quadrilaterals. The findings suggest that teacher education programs should incorporate more constructivist approaches and integrate visual materials into instructional practices to enhance pre-service teachers' knowledge of quadrilaterals. This study is expected to contribute to both the literature on quadrilateral instruction and the development of teacher training programs.

Keywords: Quadrilateral Knowledge, Van Hiele Geometric Thinking Levels, Teacher Education, Geometry Instruction, Pre-Service Elementary Mathematics Teacherstext.

1. Introduction

Human cognition naturally seeks to make sense of the environment through visual forms and patterns, making geometry a vital part of mathematical understanding. Geometry is a basic part of math that helps people understand and make sense of the space around them. It serves as a crucial link between abstract mathematical concepts and real-world applications (NCTM, 2000; Usiskin, 1995). By integrating visual representations to support mathematical coherence, geometry fosters the development of problem-solving, reasoning, and critical thinking skills, necessitating effective instruction from an early age (Sarama & Clements, 2009; Pusey, 2003; Develi & Orbay, 2003). Despite its importance, research consistently shows that

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both students and pre-service teachers encounter various conceptual challenges, particularly in recognizing, defining, and establishing relationships among quadrilaterals.

As a core topic in geometry education, quadrilaterals play a vital role in developing students' abilities to identify geometric figures, define their properties, and understand the interrelationships between different quadrilateral types (NCTM, 2000). However, literature indicates significant conceptual shortcomings in students’ understanding of quadrilaterals (Fujita & Jones, 2006a; Ayaz, 2016; Kawasaki, 1992). For instance, Ayaz (2016) found that students often rely on prototypical representations to recognize quadrilaterals but face difficulties in articulating mathematically accurate definitions, typically resorting to informal, visually based descriptions. Similarly, Kawasaki (1992) observed that many students defined rectangles incorrectly—for instance, as quadrilaterals with sides of unequal length—and that only a small fraction (approximately 5%) provided a definition that aligned with formal mathematical criteria. These findings underscore the prevalence of conceptual learning difficulties in the instruction of quadrilaterals.

Importantly, such difficulties are not confined to learners; teachers’ content knowledge significantly influences students’ conceptual understanding. The depth of teachers’ subject-matter expertise affects every stage of instruction—from lesson planning and the selection of teaching strategies to responding effectively to students’ prior knowledge (Hill, Rowan & Ball, 2005; Mewborn, 2003). Yet, several studies indicate that pre-service mathematics teachers often possess inadequate understanding of quadrilaterals (Fujita & Jones, 2007; Bütüner & Filiz, 2016; Horzum, 2018). In particular, they tend to struggle with identifying special cases of quadrilaterals, comprehending hierarchical relationships, and organizing these relationships through tools such as concept maps (Bütüner & Filiz, 2016; Horzum, 2018). Such deficiencies not only affect their own geometric reasoning but also pose a risk to the quality of students' learning experiences. Consequently, it is imperative that teacher education programs place greater emphasis on the conceptual understanding of quadrilaterals as part of a robust content knowledge framework.

Geometric thinking refers to the mental processes by which individuals understand and structure mathematical ideas through the analysis of shapes and spatial relationships (Van de Walle, 2013). Personal experiences, observation, and structured instructional opportunities shape these cognitive processes. Therefore, enhancing pre-service teachers' abilities to recognize, define, and establish meaningful connections among quadrilaterals requires fostering higher levels of geometric thinking.

The van Hiele Model is a widely accepted framework for understanding geometric thinking that conceptualizes geometric learning as a progression through five hierarchical levels (Van Hiele, 1986; Duatepe Paksu, 2016). According to this model, learners advance from one level to the next only after acquiring adequate instructional experiences at the preceding level. To ensure clarity for readers unfamiliar with the model, each van Hiele level is briefly explained below. The levels are defined as follows:

Level 0 – **Visualization**: Learners recognize and name shapes based on their overall appearance. Changes in orientation or position may hinder recognition, as the geometric properties of shapes are not yet acknowledged.

Level 1 – **Descriptive (Analysis)**: At this level, learners begin to identify and describe the properties of shapes. However, they do not yet discern relationships between shapes or recognize logical connections among these properties.

Level 2 – **Relational (Informal Deduction)**: Learners understand the hierarchical nature of geometric figures and begin to see one figure as a subset or special case of another. For instance, they recognize that a square is a specific type of rectangle.

Level 3 – **Deduction**: At this stage, students engage in formal reasoning, develop proofs, and make deductive arguments based on geometric properties and theorems.

Level 4 – **Rigor (Axiomatic Thinking)**: Learners comprehend multiple axiomatic systems, such as non-Euclidean geometries, and are able to evaluate and compare different geometric structures.

A core tenet of the van Hiele Model is that the progression through these levels is determined not by age but by the quality and nature of instructional experiences (Van Hiele, 1986). Therefore, it is essential that pre-service teachers are aware of their geometric thinking levels, both to inform their learning processes and to support the design of pedagogically effective geometry instruction for their students (Altun, 1997).

Underdeveloped geometric thinking skills can account for many of the challenges pre-service teachers face in recognizing, defining, and relating quadrilaterals (Duatepe Paksu, 2016).

The van Hiele framework provides a valuable foundation for deepening pre-service teachers' understanding of quadrilaterals, not merely at a superficial level but through structured reasoning and the recognition of hierarchical relationships. Accordingly, there is a critical need for instructional approaches that simultaneously elevate teachers’ geometric thinking and equip them with the pedagogical strategies necessary to facilitate this development in students.

The primary objective of this study is to assess pre-service elementary mathematics teachers’ knowledge of quadrilaterals and to investigate how this knowledge varies across different grade levels. In particular, the study examines teachers’ ability to define quadrilaterals based on side lengths and angle properties and their capacity to establish hierarchical relationships among quadrilateral types, all within the framework of the van Hiele geometric thinking levels. The study includes a comparative analysis of first-, second-, and third-year pre-service teachers’ quadrilateral knowledge. To this end, the research addresses the following questions:

- What is the level of pre-service teachers’ knowledge in defining quadrilaterals? Do definition skills significantly differ by grade level?
- What is the level of pre-service teachers’ ability to relate to quadrilaterals? Do relational understanding skills significantly differ by grade level?

Evaluating pre-service mathematics teachers’ quadrilateral knowledge is anticipated to yield valuable insights for enhancing the quality of teacher preparation programs. Although existing literature offers numerous studies examining teachers’ and pre-service teachers’ content knowledge in geometry, many have focused on a single quadrilateral type or a specific dimension of knowledge, such as definition or classification (Fujita & Jones, 2007; Bütüner & Filiz, 2016; Horzum, 2018). In contrast, comprehensive investigations into how pre-service teachers’ ability to define, associate, and categorize quadrilaterals during their academic years are notably limited.

This study seeks to fill that gap by offering an in-depth, comparative analysis of pre-service elementary mathematics teachers’ conceptual knowledge of quadrilaterals across different stages of their training. The findings are expected to inform both theory and practice in mathematics education by revealing patterns in the development of geometric understanding. Furthermore, the results may guide the refinement of teacher education curricula, particularly in the domain of geometry, by identifying critical content areas that require strengthened instructional emphasis. Through these contributions, the study aims to enhance scholarly debate and pedagogical practice in mathematics teacher education through these contributions.

2. Methedology

This section outlines the research design, study group, data collection instruments, data collection procedures, data analysis, and revision process of the assessment form employed in the study.

2.1. Research Design

This study utilized a cross-sectional survey model, a type of quantitative research design, to investigate pre-service elementary mathematics teachers’ knowledge of quadrilaterals based on side and angle properties. As Büyüköztürk (2016) explains, quantitative research assumes that reality is objective and independent of the researcher, enabling observable and measurable analysis. In survey models, the goal is to describe the current state of a phenomenon without researcher intervention, using representative samples rather than entire populations (Büyüköztürk et al., 2016; İslamoğlu, 2014; Yıldırım & Şimşek, 2003). Accordingly, a single administration of a structured assessment tool was used, aligning with the cross-sectional approach. As Karasar (2007) notes, this design allows for the examination of developmental differences across grade levels at a specific point in time, with the potential to infer patterns of progression.

2.2. Study Group

The study was conducted with pre-service mathematics teachers enrolled in the Elementary Mathematics Teacher Education Department. Convenience sampling was employed to ensure both accessibility and alignment with the research objectives. Participants were informed about the study’s purpose, and data were collected from those who participated voluntarily. A total of 139 pre-service teachers participated, including 105 females and 34 males. Of these, 49 were first-year (33 female, 16 male), 44 were second-year (36 female, 8 male), and 46 were third-year students (36 female, 10 male). Female participants exceeded male participants in all grade levels.

Table 1. *Distribution of Pre-Service Teachers by Grade Level and Gender*

Grade Level	Female	Male	Total
1st Year	33	16	49
2nd Year	36	8	44
3rd Year	36	10	46
Total	105	34	139

2.3. Data Collection Instrument

The creation of the data collection tool was based on a thorough review of the literature on the problems that students and pre-service teachers have when it comes to defining and classifying quadrilaterals. Specifically, studies by Fujita and Jones (2006a, 2007), Ayaz (2016), Bütüner and Filiz (2016), and Horzum (2018) provided essential insights into conceptual misunderstandings and hierarchical reasoning, which guided the selection and structuring of assessment items. To enhance validity and reliability, the content was carefully constructed and revised based on expert feedback, minimizing potential inaccuracies (Nisbet & Entswistle, 1970, as cited in Karasar, 2003). Consequently, a structured assessment form comprising 27 items was developed to evaluate pre-service teachers' knowledge of quadrilateral definitions and their ability to identify relationships among quadrilaterals.

2.4. Revision Process of the Assessment Form

The data collection instrument was designed based on the definitions provided in the curriculum and structured around research questions and the conceptual framework. To ensure its content validity and alignment with the research objectives, the instrument was reviewed by two experts specializing in mathematics education. Based on expert feedback, several specific revisions were made to improve the instrument's clarity, content validity, and alignment with the study objectives. These revisions are detailed below and include a reduction in the number of multiple-choice questions—from 27 to 12—to eliminate redundancy and enhance reliability. Additionally, the structure of the assessment was refined to target two key areas: the ability to define quadrilaterals and the ability to recognize relationships among different quadrilateral types. These changes were directly informed by expert suggestions and aim to ensure that the assessment effectively evaluates both definitional and relational understanding of quadrilaterals. These revisions are summarized as follows:

2.4.1. Reduction in the Number of Multiple-Choice Questions

Initially, the instrument contained 27 multiple-choice questions, but this number was reduced to 12. The reduction aimed to eliminate redundant questions and prevent potential reliability concerns arising from systematically repeating question formats. Questions that assessed the same conceptual understanding or followed similar question patterns were removed to ensure a more concise and reliable assessment tool.

2.4.2. Final Structure of the Assessment Form

Following the expert review and revisions, the final version of the assessment form was structured as follows:

- The first 12 questions are multiple-choice:
    - The first 6 questions assess pre-service teachers' ability to define quadrilaterals based on the relationships between their sides and angles.
    - The remaining 6 questions evaluate pre-service teachers' ability to establish relationships between different quadrilateral types.

This revised structure ensures that the assessment effectively measures both definitional knowledge and relational understanding of quadrilaterals.

2.5. Ethical

This study was conducted with the approval of the Gaziantep University Social Sciences and Humanities Research and Publication Ethics Committee with the decision dated 03/03/2025 and numbered 13..

3. Findings

Data were collected using a structured assessment form consisting of 14 items: 12 multiple-choice questions, one structured grid, and one concept map task, each targeting distinct dimensions of understanding.

The first six multiple-choice items assessed participants' knowledge of quadrilateral definitions, whereas the subsequent six examined their capacity to identify relational links among quadrilaterals. The structured grid question addressed how pre-service teachers categorize given quadrilaterals, and the concept map task evaluated their ability to construct hierarchical representations of quadrilateral relationships.

The research question "*What is the level of pre-service teachers' knowledge of quadrilateral definitions?*" was examined through six multiple-choice questions.

The descriptive statistics, including arithmetic mean, standard deviation, maximum, and minimum values, for the pre-service teachers' performance on quadrilateral definition questions are presented in Table 1. It is important to note that the results in Table 1 are based on the first six questions, with each question carrying a score of 1 point.

Table 1. *Descriptive Statistics for Pre-Service Teachers' Quadrilateral Definition Knowledge by Grade Level*

Grade Level	Arithmetic Mean	Standard Deviation	Maximum	Minimum
1st Year	4.7045	0.87815	6	3
2nd Year	4.2727	1.48424	6	1
3rd Year	4.5000	1.00000	6	1

An examination of Table 1 indicates that the mean scores for quadrilateral definition knowledge are 4.7045 for first-year, 4.2727 for second-year, and 4.5000 for third-year pre-service teachers. Given that the maximum possible score for these six questions is 6 points, it can be inferred that participants generally performed at a moderate-to-high level in defining quadrilaterals. Additionally, the first-year pre-service teachers achieved the highest average score, while second-year students exhibited slightly lower performance.

The standard deviation values show how consistent the answers are across different grade levels. A lower standard deviation indicates that responses are more closely clustered around the mean, whereas a higher standard deviation suggests greater variability in responses. Given that the range of scores in the first-year group is between 3 and 6, while for the second and third-year groups, it spans from 1 to 6, it can be inferred that first-year students demonstrated a more homogeneous distribution in their responses. Conversely, second-year students exhibited the most heterogeneous distribution, indicating greater variability in their understanding of quadrilateral definitions.

To further explore these findings, the mean, standard deviation, and percentage of correct/incorrect responses for each of the six multiple-choice questions were analyzed based on grade level. The following section presents the corresponding questions and their statistical analyses in detail.

Question 1: Identifying a Rhombus Based on Given Properties

The first question aimed to assess pre-service teachers' ability to correctly identify a rhombus using its defining properties. The given statement described a quadrilateral that has four sides, opposite sides that are parallel, and all sides of equal length. Participants were required to select the correct answer from the following options:

A) Square B) Rectangle C) Rhombus. D) Parallelogram.

The percentage of correct and incorrect responses, along with the mean and standard deviation values for each grade level, are presented in Table 2.

Table 2. *Descriptive Statistics for Question 1 (Rhombus Identification) by Grade Level*

Concept	Grade Level	Correct %	Incorrect %	Mean	Standard Deviation
Rhombus	1st Year	50.0%	50.0%	0.50	0.506
Rhombus	2nd Year	54.5%	45.5%	0.55	0.504
Rhombus	3rd Year	50.0%	50.0%	0.50	0.506
Total	-	51.5%	48.5%	0.52	0.502

As shown in Table 2, the percentage of correct answers was 50% for first-year and third-year pre-service teachers, whereas second-year students demonstrated slightly better performance with a 54.5% correct response rate. Despite this marginal difference, the mean scores were relatively close across all grade levels. Given the similarity in the standard deviation values, it can be inferred that the distribution of responses across the three groups was comparable.

An in-depth analysis of incorrect responses revealed that the majority of students who answered incorrectly mistakenly identified the given quadrilateral as a square. Specifically, 95.45% of first-year, 100% of second-year, and 100% of third-year participants who answered incorrectly selected “Square” as their response. This finding suggests that many pre-service teachers associate equal side lengths exclusively with squares, overlooking the fact that a rhombus can possess equal sides without necessarily exhibiting right angles.

Question 2: Identifying a Square Based on Given Properties

The second question required pre-service teachers to identify a square using the following defining properties: adjacent sides of equal length, all angles of equal measure, and opposite sides parallel. The available answer choices were

A) Rhombus B) Rectangle C) Parallelogram D) Square

The percentage of correct and incorrect responses, along with mean and standard deviation values, are presented in Table 3.

Table 3. *Descriptive Statistics for Question 2 (Square Identification) by Grade Level*

Concept	Grade Level	Correct %	Incorrect %	Mean	Standard Deviation
Square	1st Year	86.4%	13.6%	0.87	0.347
Square	2nd Year	68.2%	31.8%	0.68	0.471
Square	3rd Year	84.1%	15.9%	0.84	0.370
Total	-	79.5%	20.5%	0.80	0.405

As shown in Table 3, first-year pre-service teachers had the highest correct response rate (86.4%), followed closely by third-year students (84.1%), while second-year students had the lowest (68.2%), indicating greater difficulty in identifying squares. Standard deviation values suggest that first-year responses were more consistent, whereas second-year students exhibited greater variability, pointing to more diverse misconceptions. Analysis of incorrect responses revealed frequent confusion with "Rhombus" and "Parallelogram," suggesting limited differentiation based on angle properties. These findings underscore the need for instructional strategies that emphasize defining attributes over visual prototypes and support the use of hierarchical classification activities to clarify relationships among quadrilaterals.

Question 3: Identifying a Parallelogram Based on Given Properties

The third question aimed to assess whether pre-service teachers could correctly define a parallelogram based on its key attributes. The given statements included:

I. A quadrilateral in which opposite sides are parallel.
II. A quadrilateral in which all sides are of equal length.
III. A quadrilateral in which all angles are equal in measure.

Participants had to choose the correct answer from the following options:

A) Only I B) I and II C) I and III D) I, II, and III

The percentage of correct and incorrect responses, along with mean and standard deviation values, is displayed in Table 4.

Table 4. *Descriptive Statistics for Question 3 (Parallelogram Identification) by Grade Level*

Concept	Grade Level	Correct %	Incorrect %	Mean	Standard Deviation
Parallelogram	1st ear	97.7%	2.3%	0.98	0.151
Parallelogram	2nd Year	88.6%	11.4%	0.89	0.321
Parallelogram	3rd Year	93.2%	6.8%	0.93	0.255
Total	-	93.2%	6.8%	0.93	0.253

As indicated in Table 4, first-year pre-service teachers achieved the highest accuracy rate (97.7%), followed by third-year (93.2%) and second-year students (88.6%). While overall performance was strong, second-year students demonstrated greater variability (SD = 0.321), suggesting inconsistencies in understanding parallelogram properties. In contrast, first-year students' responses were the most consistent (SD = 0.151). These results imply that, although pre-service teachers generally grasp the concept of parallelograms well, some second-year students exhibit lingering misconceptions, possibly due to insufficient reinforcement of conceptual definitions across instructional levels.

Question 4: Identifying a Rectangle Based on Given Properties

The fourth question aimed to evaluate pre-service teachers' understanding of the properties of a rectangle. The given statements included:

I. A quadrilateral in which one pair of opposite sides is parallel.
II. A quadrilateral in which opposite sides are equal in length.
III. A quadrilateral in which opposite sides are parallel, all sides are equal in length, and all angles are equal in measure.

Participants were required to select the correct answer from the following options:

A) Only III B) I and II C) II and III D) I, II, and III

The percentage of correct and incorrect responses, along with mean and standard deviation values, is presented in Table 5.

Table 5. *Descriptive Statistics for Question 4 (Rectangle Identification) by Grade Level*

Concept	Grade Level	Correct %	Incorrect %	Mean	Standard Deviation
Rectangle	1st Year	70.5%	29.5%	0.71	0.462
Rectangle	2nd Year	70.5%	29.5%	0.71	0.462
Rectangle	3rd Year	86.4%	13.6%	0.86	0.347
Total	-	75.8%	24.2%	0.76	0.430

As shown in Table 5, third-year students achieved the highest accuracy rate (86.4%), while both first- and second-year students scored 70.5%. The lower performance among lower-year students indicates challenges in distinguishing rectangles from other quadrilaterals. The standard deviation values indicate that the responses from third-year students were more consistent (SD = 0.347), which suggests a stronger grasp of the concepts. On the other hand, the higher variability among first- and second-year students (SD = 0.462) suggests that they still have some misunderstandings. The frequent misidentification of rectangles suggests that there must be instructional emphasis on their defining properties rather than reliance on visual features alone.

Question 5: Identifying a Trapezoid Based on Given Properties

The fifth question assessed pre-service teachers' ability to accurately define a trapezoid using its defining characteristics. The given statements included:
I. A quadrilateral with at least one pair of parallel sides.
II. A quadrilateral in which all interior angles are equal.
III. A quadrilateral in which opposite interior angles are equal.
IV. A quadrilateral in which diagonals are equal in length.

Participants were required to select the correct answer from the following options:

A) None B) Only I C) I, II, and III D) III and IV

The percentage of correct and incorrect responses, as well as mean and standard deviation values, are presented in Table 6.

Table 6. *Descriptive Statistics for Question 5 (Trapezoid Identification) by Grade Level*

Concept	Grade Level	Correct %	Incorrect %	Mean	Standard Deviation
Trapezoid	1st Year	68.2%	31.8%	0.68	0.471
Trapezoid	2nd Year	65.9%	34.1%	0.66	0.479
Trapezoid	3rd Year	72.7%	27.3%	0.73	0.451
Total	-	68.9%	31.1%	0.69	0.465

As presented in Table 6, third-year pre-service teachers had the highest accuracy rate (72.7%), followed by first-year (68.2%) and second-year (65.9%) students, indicating a slightly stronger grasp of trapezoid properties among more advanced students. The standard deviation values were reasonably stable, with third-year students exhibiting the most consistent responses (SD = 0.451) and second-year students demonstrating the highest variability (SD = 0.479), indicating higher conceptual inconsistency in the latter group. A notable proportion of incorrect responses involved selecting "None," reflecting confusion about the basic definition of a trapezoid—specifically, the criterion of having at least one pair of parallel sides—highlighting the need for further instructional emphasis on this concept.

Question 6: Identifying a Kite Based on Given Properties

The sixth question aimed to evaluate pre-service teachers' ability to correctly define a kite by identifying its key attributes. The given statements included:

I. A quadrilateral in which all sides are of equal length.
II. A quadrilateral in which the sum of consecutive interior angles is 180°.
III. A quadrilateral in which opposite sides are parallel.
IV. A quadrilateral in which one diagonal divides the shape into two isosceles triangles.
V. A quadrilateral in which at least one pair of opposite angles is equal.

Participants were required to choose the correct answer from the following options:

A) Only I B) I, II, and IV C) I and V D) IV and V

The percentage of correct and incorrect responses, along with mean and standard deviation values, is displayed in Table 7.

Table 7. *Descriptive Statistics for Question 6 (Kite Identification) by Grade Level*

Concept	Grade Level	Correct %	Incorrect %	Mean	Standard Deviation
Kite	1st Year	97.7%	2.3%	0.98	0.151
Kite	2nd Year	79.5%	20.5%	0.80	0.408
Kite	3rd Year	63.6%	36.4%	0.64	0.487
Total	-	80.3%	19.7%	0.80	0.399

As shown in Table 7, first-year pre-service teachers had the highest accuracy rate on the kite question (97.7%), followed by second-year (79.5%) and third-year students (63.6%), indicating a decline in conceptual retention over time. Standard deviation values revealed that responses were most consistent among first-years (SD = 0.151) and most varied among third-years (SD = 0.487), suggesting greater misconceptions at higher levels. Incorrect responses showed that many second- and third-year students confused kites with rhombuses, incorrectly endorsing properties like equal side lengths and 180° angle sums. This points to a need for a clearer distinction between quadrilateral types in instruction.

Overall, the parallelogram question had the highest accuracy rate across all levels (93.2%), while the rhombus question had the lowest (51.5%). Each grade level performed best on the parallelogram question and worst on the rhombus question. These findings suggest that familiarity and distractors impact accuracy, highlighting the importance of reinforcing hierarchical relationships and precise definitions in teacher education

To determine whether quadrilateral identification knowledge significantly differed across class levels, a one-way analysis of variance (ANOVA) was conducted for each question and the total identification score. The mean scores and ANOVA results for class-level comparisons are summarized in Table 8.

Table 8. *Comparison of Quadrilateral Identification Total Scores by Class Level*

Grade Level	N	Mean ± SD	Levene's Test	ANOVA-F	p-value
1st Year	44	4.70 ± 0.88	p < 0.05	1.55	0.216
2nd Year	44	4.27 ± 1.48	-	-	-
3rd Year	44	4.50 ± 1.00	-	-	-

As presented in Table 8, quadrilateral identification scores across all grade levels were above the mid-range, reflecting a generally moderate to high level of conceptual understanding. First-year students had the highest average score (M = 4.70), followed by third-year (M = 4.50), with second-year scoring the lowest (M = 4.27).

However, one-way ANOVA results indicated no statistically significant differences among grade levels ($p > .05$). This suggests that performance variations are not systematically linked to grade progression and may stem from individual learning differences. The absence of a consistent trend indicates the need for structured and progressive instructional support to enhance and sustain pre-service teachers' understanding of quadrilateral definitions. These findings reinforce previous research emphasizing the importance of targeted pedagogical interventions in geometry education.

In the final six multiple-choice questions, pre-service teachers were assessed on their knowledge of the relationships between different quadrilateral types. These questions aimed to answer the research question: "How well do pre-service teachers understand the relationships between quadrilaterals?" The findings for these six questions are reported across first-, second-, and third-year cohorts. Since each of these six questions carried a maximum score of 1, the mean scores, standard deviations, and minimum-maximum values across class levels are presented in Table 9.

Table 9. *Descriptive Statistics for Quadrilateral Relationship Knowledge Across Class Levels*

Grade Level	Mean	Standard Deviation	Maximum	Minimum
1st Year	5.1591	0.74532	6	3
2nd Year	4.8636	0.95457	6	3
3rd Year	5.0682	0.78940	6	3

First-year pre-service teachers attained the highest mean score (5.16), followed by third-year (5.07) and second-year (4.86) students. Considering the maximum score was 6, overall performance across all groups indicates a high level of achievement. However, the lower mean for second-year students suggests a slight decline in performance. Score distributions showed a minimum of 3 and a maximum of 6 across all groups, indicating that even the lowest scores reflected a moderate understanding of quadrilateral relationships. Standard deviation values revealed that first-year students' responses were the most consistent, while second-year responses were more varied, indicating greater heterogeneity in their understanding. To explore these findings further, the final six multiple-choice items were analyzed for accuracy by grade level. The seventh question, focusing on a unique property of a rhombus not shared by all parallelograms, is summarized in Table 10.

Table 10. *Accuracy Rates, Mean Scores, and Standard Deviations for Question 7 (Rhombus-Parallelogram Relationship)*

Concept	Grade Level	Correct %	Incorrect/Unanswered %	Mean	Standard Deviation
Rhombus-Parallelogram	1st Year	88.6%	11.4%	0.89	0.321
	2nd Year	75.0%	25.0%	0.75	0.438
	3rd Year	90.9%	9.1%	0.91	0.291
	Overall	84.8%	15.2%	0.85	0.360

As shown in Table 10, first-year (88.6%) and third-year (90.9%) students demonstrated the highest accuracy rates, while second-year students exhibited the lowest accuracy (75.0%). Third-year students' responses were the most consistent, as evidenced by the lowest standard deviation (0.291), whereas second-year students' answers showed greater variability (0.438).

The eighth question examined pre-service teachers' understanding of a property that is unique to squares and does not apply to all rectangles. The distribution of accuracy rates, means, and standard deviations is presented in Table 11.

Table 11. *Accuracy Rates, Mean Scores, and Standard Deviations for Question 8 (Square-Rectangle Relationship)*

Concept	Grade Level	Correct %	Incorrect/Unanswered %	Mean	Standard Deviation
Square-Rectangle	1st Year	93.2%	6.8%	0.93	0.255
	2nd Year	95.5%	4.5%	0.95	0.211
	3rd Year	95.5%	4.5%	0.95	0.211
	Overall	94.7%	5.3%	0.95	0.225

As seen in Table 11, pre-service teachers across all grade levels demonstrated high accuracy rates (above 93%), with second- and third-year students achieving identical accuracy rates of 95.5%. Standard deviation values indicate that responses were relatively homogeneous across groups, reflecting a strong and uniform understanding of this relationship. The ninth question focused on differentiating squares from rhombuses by identifying a property exclusive to squares. The distribution of correct and incorrect responses for this question is displayed in Table 12.

Table 12. *Accuracy Rates, Mean Scores, and Standard Deviations for Question 9 (Square-Rhombus Relationship)*

Concept	Grade Level	Correct %	Incorrect/Unanswered %	Mean	Standard Deviation
Square-Rhombus	1st Year	93.2%	6.8%	0.93	0.255
	2nd Year	84.1%	15.9%	0.84	0.370
	3rd Year	86.4%	13.6%	0.86	0.347
	Overall	87.9%	12.1%	0.88	0.328

As shown in Table 12, first-year students had the highest accuracy rate (93.2%), followed by third-year (86.4%) and second-year (84.1%) students. The lower performance among second-years suggests some difficulty in distinguishing square-specific properties from those of rhombuses. Standard deviations indicate that first-year responses were the most consistent, while second-year responses were more variable. Overall, pre-service teachers had a good understanding of how quadrilaterals relate to each other, especially when it came to telling squares apart from rectangles and rhombuses. However, the variability among second-year students points to gaps in conceptual consistency. These findings highlight the need to reinforce hierarchical classification of quadrilaterals within teacher education programs to ensure more coherent conceptual development. Table 13. presents the results of the tenth question, which assessed understanding of hierarchical relationships among quadrilaterals.

Table 13. *Accuracy Rates, Mean Scores, and Standard Deviations for Question 10 (General Quadrilateral Comparisons)*

Concept	Grade Level	Correct %	Incorrect/Unanswered %	Mean	Standard Deviation
General Comparison	1st Year	95.5%	4.5%	0.96	0.211
	2ndYear	95.5%	4.5%	0.96	0.211
	3rd Year	86.4%	13.6%	0.86	0.347
	Overall	92.4%	7.6%	0.92	0.266

As shown in Table 13, first- and second-year students exhibited the highest accuracy rates (95.5%), while third-year students showed a lower accuracy rate (86.4%). Standard deviation values indicate that responses from first- and second-year students were more consistent (0.211), whereas third-year students exhibited more variability in their responses (0.347).

The eleventh question examined pre-service teachers' understanding of the relationships among rhombuses, parallelograms, rectangles, and kites. The accuracy rates, means, and standard deviations are presented in Table 14.

Table 14. *Accuracy Rates, Mean Scores, and Standard Deviations for Question 11 (General Quadrilateral Comparisons)*

Concept	Grade Level	Correct %	Incorrect/Unanswered %	Mean	Standard deviation
General Comparison	1st Year	% 95.5	4.5%	0.96	0.211
	2nd Year	%88.6%	11.4%	0.89	0.321
	3rd Year	%72.7	27.3%	0.73	0.451
	Overall	% 85.6%	14.4%	0.86	0.352

According to Table 14, first-year students achieved the highest accuracy rate (95.5%), followed by second-year students (88.6%) and third-year students (72.7%). These results suggest that as students progress through their teacher education program, their conceptual understanding of quadrilateral relationships may become less stable. In later years of study, an increasing reliance on procedural knowledge rather than conceptual understanding could be the cause of this decline.

Furthermore, standard deviation values indicate greater response variability among third-year students (0.451), while first-year students' responses were the most homogeneous (0.211). The increasing variability among upper-year students suggests that some pre-service teachers may struggle to consolidate their foundational knowledge of quadrilateral classification and hierarchical relationships.

Overall, the findings highlight that pre-service teachers generally demonstrate a strong understanding of quadrilateral relationships, particularly in the earlier stages of their education. However, variability in responses among third-year students suggests potential gaps in the retention and deeper conceptualization of these relationships. These results indicate a need for a more structured and cumulative approach to teaching hierarchical classifications of quadrilaterals, ensuring that pre-service teachers develop a coherent and robust understanding that persists throughout their training.

As shown in Table 15, the twelfth question was correctly answered by 50% of first-year pre-service teachers, 47.7% of second-year pre-service teachers, and 75% of third-year pre-service teachers. Third-year students demonstrated the highest accuracy rate on this question, while second-year students had the lowest. The standard deviation values were 0.506 for first-year students, 0.505 for second-year students, and 0.438 for third-year students, indicating that third-year students' responses were more consistent compared to those of first- and second-year students.

Table 15. *Accuracy Rates, Mean Scores, and Standard Deviations for Question 12 (General Quadrilateral Comparisons)*

Concept	Grade Level	Correct %	Incorrect/Unanswered %	Mean	Standard Deviation
General Comparison	1st Year	50.0%	50.0%	0.50	0.506
	2nd Year	47.7%	52.3%	0.48	0.505
	3rd Year	75.0%	25.0%	0.75	0.438
	Overall	57.6%	42.4%	0.58	0.496

A comparative analysis of pre-service teachers' responses to the last six multiple-choice questions revealed that Question 8 had the highest accuracy rate (94.7%), whereas Question 12 had the lowest (57.6%). The results suggest that Question 12 posed greater conceptual difficulties for students across all grade levels. Additionally, in all quadrilateral relationship questions except for Question 12, more than 50% of students in each grade level provided correct answers. In contrast, less than 50% of second-year students answered Question 12 correctly, indicating a relative weakness in this area.

The differing accuracy rates for different questions may be due to students' different levels of understanding of how quadrilaterals relate to each other or the effect of distractors in multiple-choice options. This finding indicates that pre-service teachers continue to hold certain misconceptions about the classification and hierarchical properties of quadrilaterals.

A one-way analysis of variance (ANOVA) was conducted to determine whether pre-service teachers' ability to establish relationships between quadrilaterals differed significantly across grade levels. Table 16 presents the mean scores and ANOVA results for each grade level.

Table 16. *Comparison of Quadrilateral Relationship Scores Across Grade Levels*

Grade Level	n	M (SD)	Levene's Test	F	p
1st Year	44	5.16 (0.75)	$p < .05$	1.45	.239
2nd Year	44	4.86 (0.95)			
3rd Year	44	5.07 (0.79)			

The results in Table 16. show that mean scores for quadrilateral relationship knowledge were above the mid-range for all grade levels. First-year students had the highest mean score (5.16), while second-year students had the lowest mean score (4.86). Despite this variation, ANOVA results revealed no statistically significant difference between grade levels ($p > 0.05$).

This finding suggests that, although first-year students achieved slightly higher scores on average, the differences among grade levels were not statistically significant. This implies that progression through the teacher education program does not necessarily lead to significant improvement in pre-service teachers' ability to relate quadrilateral properties.

4. Discussion and Conclusion

This study examined the knowledge of pre-service elementary mathematics teachers regarding the definition and classification of quadrilaterals, focusing on whether their understanding differed by grade level. Additionally, misconceptions related to quadrilaterals, pre-service teachers' van Hiele geometric thinking levels, and their ability to establish relationships between quadrilaterals were analyzed. The results were analyzed in conjunction with existing literature to enhance comprehension of the findings.

The analysis of pre-service teachers' ability to define quadrilaterals revealed no statistically significant differences across grade levels. Although first-year students had slightly higher mean scores ($M = 4.70$) than second-year ($M = 4.27$) and third-year students ($M = 4.5$), this difference was not statistically significant ($p > .05$). Effect size calculations (Cohen’s d) also indicated a small magnitude of difference, suggesting that the observed variation is unlikely to have practical or pedagogical significance. Overall, the results suggest that pre-service teachers at different grade levels have about the same amount of knowledge about the definitions of quadrilaterals.

According to the multiple-choice test results on quadrilateral definitions, the parallelogram was the most accurately identified shape (93.2%), while rhombus had the lowest correct response rate (51.5%). This indicates that pre-service teachers can more easily recognize and define quadrilaterals that are frequently encountered, such as squares and parallelograms, but struggle with special cases like rhombuses. This finding aligns with the observations of Fujita and Jones (2006), who indicated that students and pre-service teachers predominantly depend on prototypical representations when defining quadrilaterals.

A closer analysis of incorrect responses indicated that many pre-service teachers confused squares and rhombuses, suggesting that they perceive quadrilaterals as distinct entities rather than understanding them within a hierarchical classification system. This observation is consistent with Okazaki and Fujita (2007), who found that students often lack a strong conceptual understanding of shared quadrilateral properties and struggle with relational reasoning. Therefore, explicit instruction on hierarchical structuring of quadrilateral properties is essential for deeper conceptual learning.

Regarding pre-service teachers' ability to classify quadrilaterals and recognize relationships between them, the results indicate a high level of understanding overall. However, defining special cases and establishing hierarchical relationships remain areas of difficulty. For instance, 90.9% of third-year pre-service teachers got Question 7 right, which asked about the relationship between rhombuses and parallelograms. Only 75.0% of second-year pre-service teachers got it right. This implies that as pre-service teachers advance in their education, they enhance their comprehension of certain quadrilateral relationships. However, no statistically significant differences were found across grade levels in general, implying that the learning process for hierarchical quadrilateral classification extends over time and cannot be explained solely by grade level.

In Question 8 (relationship between squares and rectangles), all grade levels had a high accuracy rate (94.7%), whereas Question 12 (general quadrilateral classification) had the lowest accuracy rate (57.6%). This suggests that pre-service teachers excel at recognizing direct relationships between specific quadrilaterals but struggle with establishing an overarching conceptual hierarchy. This aligns with findings from Erşen and Karakuş (2013), who observed that pre-service teachers are generally better at defining individual quadrilaterals but face difficulties in recognizing connections between them.

Another notable finding is that, in general comparison questions (Questions 10 and 11), first-year pre-service teachers exhibited higher accuracy rates than their peers in other grade levels. However, in Question 12, third-year pre-service teachers achieved the highest success rate. This suggests that while first-year students may perform better in some specific classification tasks, the ability to construct conceptual relationships among quadrilaterals improves over time. These results indicate that there is a stronger emphasis on quadrilateral relationships in teacher education programs.

Examining the results through the Van Hiele geometric thinking framework, it appears that most pre-service teachers operate at Level 2 (Descriptive/Analytical) while struggling to transition to Level 3 (Relational

Deduction). The lower-than-expected geometric thinking levels align with previous studies conducted by Şahin (2008) and İlhan (2011), which reported that pre-service teachers tend to analyze quadrilaterals individually rather than integrating them into a hierarchical structure.

To improve pre-service teachers' geometric thinking levels, teacher education programs should incorporate more constructivist approaches, such as problem-based learning and guided discovery activities. These methods can be supported by the use of dynamic geometry software (e.g., GeoGebra or Cabri) and interactive instructional techniques that encourage exploration and reasoning. Implementing such practices across multiple semesters—particularly in content-focused courses during the first and second years—can provide sustained opportunities for conceptual development and reflection.Research by Sancar (2019) demonstrated that using concept cartoons in teaching geometry fosters positive attitudes and enhances conceptual understanding. Similarly, Balgalmış and Işık (2019) found that relationship-based instruction is more effective in helping students establish connections among quadrilaterals. Therefore, instructional activities must not solely concentrate on defining quadrilaterals but also on facilitating pre-service teachers' comprehension of their interrelations.

The present study contributes to the literature by highlighting the conceptual difficulties pre-service elementary mathematics teachers face in defining and classifying quadrilaterals. Despite the absence of statistically significant differences across grade levels, misconceptions continued to persist in all groups. Therefore, teacher education programs should place greater emphasis on developing geometric thinking through constructivist, technology-supported, and conceptually focused instruction. Recent studies provide strong evidence supporting this approach. For instance, Elbehary (2023) showed that van Hiele-based instruction supported by Geometer's Sketchpad significantly improved pre-service teachers' content knowledge and attitudes toward technology. Abidin and Abu (2021) reported that instructional videos based on the van Hiele model positively affected students' geometric thinking levels. Moreover, Bonyah (2021) emphasized the model's applicability across diverse learner profiles by showing that pre-service teachers exhibited varied geometric thinking levels, with many demonstrating limited reasoning even at lower van Hiele levels. These findings suggest that adopting multimodal, learner-centered approaches grounded in van Hiele theory can promote inclusive and effective geometry instruction.

5. Recommendations

To enhance pre-service teachers' understanding of quadrilaterals, the study suggests incorporating constructivist approaches, utilizing dynamic geometry software, and implementing teaching strategies that explicitly support hierarchical classification skills. Instruction should emphasize not only defining quadrilaterals but also recognizing their interrelations within a systematic framework.

This research contributes significantly to the field of mathematics education by identifying the current state of pre-service teachers' quadrilateral knowledge and discussing how instructional practices can be optimized for more effective teaching. Future studies should focus on designing innovative instructional strategies that enhance pre-service teachers' learning processes related to quadrilateral classification and hierarchical reasoning.

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