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Analysis of a Second-Order Factor Model of Attitude Toward Mathematics*

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ABSTRACT

Based on a multidimensional definition of attitude, this study tested a second-order factor model of attitude toward mathematics using data from 1960 7th-grade students. This study found that students' attitudes toward mathematics are identified with cognitive, affective, and behavioral factors. This study confirmed three hypotheses: the cognitive component of attitude toward mathematics is explained by students' confidence in learning mathematics and their beliefs about the usefulness and importance of mathematics; the affective component of attitude toward mathematics is explained by their liking of mathematics and mathematics anxiety; and the behavioral component of attitude toward mathematics is explained by learner behaviors toward mathematics and the time they spent on mathematics at home. Moreover, the final model obtained through factor analysis revealed strong relationships between the three components of attitude and attitude toward mathematics.

Keywords:
Attitude toward mathematics, measurement model, components of attitude, elementary school students

1. Introduction

1.1. Attitude in Mathematics Education

In mathematics education within the field of affect, research on attitude has a long history of being investigated through ambiguous theoretical frameworks (Di Martino & Zan, 2011). Early studies on attitude mainly aimed to identify causal correlations between attitude and other variables (in particular, mathematical achievement). These mainly focused on measurement instruments rather than the theoretical clarification of the construct (Di Martino & Zan, 2015) and may have resulted in hindering the development of an adequate theory (Kulm, 1980; Leder, 1985; McLeod, 1992; Ruffell, Mason, & Allen, 1998). However, numerous resulting studies lacked clear evidence of a correlation between attitude and achievement in mathematics. These revealed the need for clarifying the definition of attitude (Aiken, 1970). Thereby, recognizing the seminal aspect of affective issues in mathematics learning and instruction, the construct of attitude regained significance as a field of research.

Theoretical discourse regarding research on attitude necessarily begins with a 'definition problem' (Di Martino & Zan, 2015). Early studies generally described attitude as a predisposition to respond to a certain object either positively or negatively (Haladyna, Shaughnessy, & Shaughnessy, 1983; McLeod, 1992). However, this mono-dimensional definition ignores attitude-related cognitive elements (Hannula, 2002). Neale (1969) expanded on the notion of attitude by considering various domains associated with attitude. He defined mathematics attitude through the characteristics of *liking or disliking* mathematics, *engaging in or avoiding* mathematical activities, personal *belief* regarding one's *ability* in mathematics, and belief regarding mathematics' *usefulness*

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or uselessness. It is now acknowledged by the most recent theories in social psychology that attitude is a multidimensional construct having cognitive, affective, and behavioral components (Eagly & Chaiken, 1998). Within the field of mathematics education, many explicit definitions of attitude refer to this multidimensional definition (Di Martino & Zan, 2015), describing attitude using three components: the emotional disposition towards mathematics, the set of beliefs regarding mathematics, and the behavior related to mathematics (Hart, 1989; Leder, 1992; Ruffell et al., 1998). The theoretical framework of attitude used in this study builds on this multidimensional definition and recognizes attitude's cognitive, affective, and behavioral components. In other words, "a positive or negative attitude toward mathematics could be inferred from one's emotional reaction to mathematics, one's behavior in approaching or avoiding mathematics, and one's belief about what mathematics is and how it may be used" (Hart, 1989, p. 39).

1.2. Beliefs, Emotions, and Behavior

While *belief* is frequently discussed in mathematics education, Furinghetti and Pehkonen (2002) realized the field's lack of agreement on what beliefs are. However, Thompson (1992) opines that there is some agreement that mathematical beliefs are considered personal philosophies or conceptions about the nature of mathematics and its teaching and learning.

Beliefs are not independent of each other; they occur in clusters (Green, 1971). It is possible to classify mathematics-related beliefs in several ways. While various researchers differ in their categorizations, there is broad agreement regarding three dimensions of beliefs about mathematics: beliefs about mathematics education, beliefs about self, and beliefs about the social context (Op't Eynde, de Corte, & Verschaffel, 2003). Since each category has several associated subcategories and it would be unfeasible to include them all in a single study, this study opts to consider beliefs about the nature of mathematics, learning mathematics (i.e., the usefulness and importance of mathematics), and beliefs about oneself (i.e., confidence in learning mathematics).

Despite being arguably the most fundamental concept, *emotion* has been used less in mathematics education research (compared to attitudes and beliefs). While cognitive constructivists suggest that repeated emotional experiences can stabilize attitudes and beliefs (Zan et al., 2006), repeated emotional reactions to mathematical situations can become habituated in attitudes toward mathematics (McLeod, 1992). Despite the different approaches to emotions in mathematics education (Evans & Zan, 2006), there is some agreement: emotions are seen to involve psychological reactions and affect cognitive processes in several ways (biasing attention and memory and activating action tendencies) and are recognized as functional, with a key role in human coping and adaptation (De Bellis & Goldin, 2006; Evans, 2000).

According to Hannula (2012, p. 143), there is general agreement that emotions consist of three processes: "psychological processes that regulate the body, subjective experiences that regulate behavior, and expressive processes that regulate social coordination" (Buck, 1999; Power & Dangleish, 2015). This paper follows the premise of enumerating only a few basic emotions of happiness, sadness, fear, anger, disgust, and interest and recognizes that more complex emotions are constructed upon these (Buck, 1999; Power & Dangleish, 2015). Among the basic emotions, this study limited its focus to liking mathematics and mathematics anxiety as embodying a student's emotions regarding mathematics.

The construct of attitude, shaped by the context of social psychology, orients certain *behaviors*. Thus, a bidirectional path is formed such that (1) an understanding of attitude leads to the possible prediction of behavior and (2) observing behavior provides a window to attitude (Di Martino & Zan, 2011). This study focuses on the first direction of this path and, to assess the behavioral component of attitude toward mathematics, considers students' behavior in mathematics class, such as attending classes, raising hands, and doing homework, and the amount of time he or she spends on mathematics at home.

1.3. The Measurement of Attitude

As previously mentioned, early studies on attitude primarily focused on measuring the degree of one's liking or disliking of mathematics. Later paper-and-pencil scales developed by mathematics educators demonstrated a multidimensional view of attitudes toward mathematics (e.g., scales from the National Longitudinal Study of Mathematical Abilities [Crosswhite, 1972], the Mathematics Attitude Inventory [Sandman, 1980], and the most extensively used in research studies, the Fennema-Sherman Mathematics Attitude Scales (FSMAS)

[Fennema & Sherman, 1976]) (Hart, 1989). More recently, Hannula (2011) reviewed several survey instruments' dimensions of the mathematics-related affect of several survey instruments. Among the nine scales of the FSMAS, he classified the confidence in learning mathematics scale into beliefs about self; the mathematics usefulness scale into beliefs about teaching and learning; the mother scale, the father scale, and the teacher scale into beliefs about social context; the mathematics anxiety scale into emotional traits; and the attitude toward success in mathematics scale, the mathematics as a male domain scale, and the effectance motivation scale in mathematics into motivations (including values). Since FSMAS relies on participants' responses to a series of statements about mathematics learning and enables measurement congruent with the cognitive and affective components of attitude toward mathematics, it was found to have a functional toll for this investigation. Historically, psychometric studies of the FSMAS have found agreement on the reliability and validity of the scales (e.g., Melancon, Thompson, & Becnel, 1994; Mulhern & Rae, 1998).

Methodological criticism of research on attitude considers the inflexibility of the traditional assessment instruments measuring attitudes with often competing or inexplicit definitions of attitude (Di Martino & Zan, 2015). The gap between the definition of attitude and its measurement (Leder, 1985) results in a lack of reliability of the observational instruments (Di Martino & Zan, 2015). Further critiques regarding attitude research recognize that instruments measuring attitude do so through the summing of the three-fold components of cognitive, affective, and behavioral, and this introduces the problem of the correlations among measures of the three components of considerable magnitude (Ajzen, 1988). Indeed, it is acknowledged that the three components are closely related (Triandis, 1971).

In light of these critical aspects, this study explicitly defines attitude and addresses the coherency between the definition and the instruments used to measure it. Some extant longitudinal research on math attitude and performance has demonstrated the relationships of variables associated with math attitude through path analyses (Abu-Hilal, 2000; Eccles & Jacobs, 1986; Meece, Wigfield, & Eccles, 1990; Pajares & Miller, 1994). Additionally, increased computational capacity has facilitated more advanced quantitative methods, such as meta-analysis and Structural Equation Models, to test more complex hypotheses with data from large-scale surveys (Hannula, 2011). Therefore, this study employs a second-order factor model of attitude toward mathematics to draw a clear picture of attitude with its components. The specific research question of the study is: What is the factor model explaining the three components of attitude toward mathematics?

2. Methodology

2.1. Participants

The study sample consisted of 1960 7th-grade students enrolled in 19 elementary schools in Istanbul, Turkey. Convenience sampling was used to select students based on a school location and one grade (7th). There were 1001 (51.1%) females and 959 (48.9%) males. The average student's previous year's school grade in the mathematics course was 3.15 over 5.00. Regarding students' parents' educational level, 54.2% of their mothers and 44.3% of their fathers were primary school graduates. Moreover, 45.3% of the students had no help with math from outside of school. While student gender, parent education level, etc. were not part of the study, this data was collected for potential future analysis.

2.2. Instrument

To model the factor structure of students' attitudes toward mathematics, the Attitudes Towards Mathematics Questionnaire (ATMQ) was formed. The ATMQ consisted of two parts. The first part included six questions related to the information about students' demographic characteristics, such as school name, class, gender, parent's education level, previous year's school grade in the mathematics course, and the type of help students get for mathematics outside of school. In the second part, there were seven scales: Confidence in Learning Mathematics Scale (CONF), Usefulness of Mathematics Scale (USE), Importance of Mathematics Scale (IMP), Liking of Mathematics Scale (LIKE), Mathematics Anxiety Scale (ANX), Learner Behaviors toward Mathematics Scale (MATBEHA), and Time Spent on Mathematics at Home Scale (TIME). CONF, USE, and IMP were used to measure observed cognitive components of attitude toward mathematics; LIKE and ANX were used to measure observed affective components of attitude toward mathematics; MATBEHA and TIME were used to measure observed behavioral components of attitude toward mathematics. As the literature shows, these three factors would most likely indicate a second-order factor: attitude.

The Confidence in Learning Mathematics, Usefulness of Mathematics, and Mathematics Anxiety scales were adapted previously from the FSMAS (1976) corresponding scales by Tag (2000). The Importance of Mathematics scale was adapted from TIMSS (1999) previously by Tag (2000). The Liking of Mathematics, Learner Behaviors toward Mathematics, and Time Spent on Mathematics at Home scales were translated and adapted by the researchers from the instruments of TIMSS (1999), Neustadt's (2005), and Mohamad-Ali's (1995), respectively. While putting the statements on the scales adapted by Tag, each statement was checked separately to see if some statements may cause any misunderstanding or misinterpretation by the students. After determining the problematic statements, the instruments were reconstructed by the researchers. Moreover, the statements selected from the instruments of TIMSS (1999), Neustadt's (2005), and Mohamad-Ali's (1995) were translated into Turkish by researchers and an expert in Foreign Language Education. Then the researcher translated the statements back into English, and the original statements were compared with the adapted ones.

Altogether, the literature and previous studies and scales led to the following variables for this study:

- **Confidence in Learning Mathematics (CONF):** It refers to students' beliefs about their ability to learn and perform well on mathematical tasks. The dimensions range from a distinct lack of confidence to definite confidence.
- **Usefulness of Mathematics (USE):** It refers to students' beliefs about the usefulness of mathematics currently and in relation to their future education, vocation, or other activities.
- **Importance of Mathematics (IMP):** It refers to students' beliefs about the importance of mathematics in relation to their lives.
- **Liking of Mathematics (LIKE):** It refers to a student's liking or disliking of mathematics and his or her positive and negative feelings about mathematics or himself or herself as a mathematics learner.
- **Mathematics Anxiety (ANX):** It refers to feelings of anxiety, dread, nervousness, and associated bodily symptoms related to doing mathematics. The dimensions range from feeling at ease to those distinct anxieties.
- **Learner Behaviors toward Mathematics (MATBEHA):** It refers to ways students learn mathematics (learning style), participation in math classes, hand rising, and doing mathematics homework.
- **Time Spent on Mathematics at Home (TIME):** It refers to students' estimates of the time they spent on mathematics at home.

2.3. Data Analysis

The reliability of the subscales was assessed using Cronbach alpha coefficients. The underlying structure of the ATMQ was explored using principal components analysis (PCA). It was used to extract the factors followed by oblique rotation of factors using oblimin rotation (delta = 0) by using SPSS version 28. PCA is a form of factor analysis that is commonly used by researchers interested in scale development and evaluation (Pallant, 2020). Moreover, it was adopted by Melancon et al. (1994) and Mulhern and Rae (1998) in their study of the factor validity of the FSMAS. The number of factors to be retained was guided by two decision rules of Kaiser's criterion (eigenvalues above 1) and inspection of the screeplot.

Second-order confirmatory factor analysis (CFA) was conducted using LISREL version 8.30 (Jöreskog & Sörbom, 1999). The general sequence of CFA-based higher factor analysis is as follows: (1) Develop a well-behaved first-order CFA solution; (2) examine the magnitude and pattern of correlations among factors in the first-order solution; and (3) fit the second-order factor model, as justified on conceptual and empirical grounds (Brown, 2015).

Listwise deletion of cases was used for missing data in all analyses. The multivariate normality and linearity of scales were evaluated through SPSS and LISREL. The data from a few students with low scores on some scales was kept in the analysis. Using Mahalanobis distance, twenty students were multivariate outliers, $p < .001$, and the data from these students were kept. The normality tests showed sufficient evidence that the assumption of a multivariate normal data distribution might not be violated. Therefore, maximum likelihood estimation was used to fit the models to the data set. While deciding regarding data-model fit, the chi-square test, along with its degrees of freedom and p-values, and fit indices (Standardized Root Mean Squared Residual (SRMR), Root-Mean-Square Error of Approximation (RMSEA), Comparative Fit Index (CFI), and Non-Normed Fit Index (NNFI)) were reported. SRMR values of .08 or less are indicative of a good fit; the RMSEA index should be .05 or below for a well-fitting model, or .08 or below for an acceptable model; the values of the CFI and NNFI indices should be .95 or above (Bandalos & Finney, 2010).

2.4. Ethical

Since it is a study that includes data scanning, there is no need to obtain permission from any ethics committee.

3. Findings

3.1. Descriptive Statistics

In the present study, the results of the descriptive statistics of the scales with 1960 7th-grade students are given in Table 1.

Table 1. *Scales, Minimums and Maximums, Means, Standard Deviations, Medians*

Scale	Min.	Max.	M	*SD*	Mdn
CONF	13	60	43.44	9	43
USE	18	60	48.71	8.05	50
IMP	5	25	20.12	4.01	21
LIKE	5	25	19.14	4.18	20
ANX	12	60	41.37	9.25	41
MATBEHA	6	20	16.52	2.86	17
TIME	4	20	13.19	3.65	13

Descriptive statistics indicated that the mean scores of the scales were above the midpoint of the range and close to the highest end of the continuum. Thus, these students believed they had definite confidence rather than a distinct lack of confidence in learning mathematics. They were more likely to view mathematics as a useful field of study currently and in relation to their future education, vocation, or other activities, as well as as important in their lives. They enjoyed mathematics and had feelings of ease related to doing mathematics. They were generally actively involved in math classes, such as attending classes, raising hands, and doing homework, and were likely to spend their time on mathematics at home.

3.2. Reliability

The Cronbach alpha values for CONF, USE, IMP, LIKE, ANX, MATBEHA, and TIME were .88, .85, .78, .77, .83, .45, and .66, respectively. The value of .45 for the MATBEHA required extreme caution. After analyzing the items in the MATBEHA separately, it was found that the removal of item 5 (*"I am more likely to raise my hand for a question than an answer"*) would improve the Cronbach alpha value to .60. The value of the corrected-item total correlation of item 5 was very low (-.03). For reliability purposes, it was removed. Most of the Cronbach alpha values exceed the recommended value of .7 (Nunnally, 1978), indicating adequate internal consistency. Moreover, the values can be quite small when a small number of items are on the scale (Pallant, 2020).

3.3. Principal Components Analysis

The sample was first assessed for its suitability for PCA. Inspection of the correlation matrix revealed the presence of many coefficients of .3 and above. Bartlett's Test of Sphericity was highly significant ($p < .001$) and the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy value of .9 exceeded the recommended value of .6 (Tabachnick & Fidell, 2013), supporting the factorability of the matrix.

PCA revealed the presence of ten components with eigenvalues exceeding 1, explaining 26.2%, 7.9%, 6.6%, 3.1%, 2.8%, 2.3%, 2.2%, 2.1%, 2.0%, and 1.9% of the variance, respectively. Only these first ten factors exceeded the criterion value obtained from the parallel analysis. An inspection of the screeplot revealed a clear break after the third component and a lesser break after the fifth component. Thus, the number of components was decided after inspecting the pattern matrix. Inspection of the pattern matrix (Table 2) showed a relatively clear ten-factor solution. The interpretation of the ten components was consistent with Tag's research on CONF, USE, IMP, and ANX scales, with positive and negative items loading strongly on separate factors for each scale. One LIKE item (LIKE5) loaded strongly (.413) and inappropriately onto the IMP factor and weakly loaded (-.231) on the LIKE factor. One ANX item (ANX2) also showed crossloading on the LIKE factor (-.393), and it showed a lower loading on the ANX factor (.145).

Analysis of the structure matrix (Table 3) indicated good discrimination between the factors. For each component, the lowest factor loading for its items was higher than the highest loading on that factor of other items (except for LIKE5 and ANX2).

Overall, the results of the analysis showed that ATMQ has ten subdimensions, which are CONF_P (CONF with positive items), CONF_N (CONF with negative items), USE_P (USE with positive items), USE_N (USE with negative items), IMP, LIKE, ANX_P (ANX with positive items), ANX_N (ANX with negative items), MATBEHA, and TIME. However, the contents of the factors obtained do not fully support the original structures of the LIKE and ANX scales proposed by Tag. The major inconsistency concerning this sample is the tendency of one of the LIKE items (LIKE5: "*I need to do well in mathematics to please myself*") to show more substantial loadings on the IMP factor, and one of the ANX items (ANX2: "*It wouldn't bother me at all to take more math courses*") to show more substantial loadings on the LIKE factor. It was therefore decided to explore the structure of the ATMQ with these two items removed.

PCA with oblimin rotation was repeated with LIKE5 and ANX2 removed from their respective scales. This resulted in a 52-item scale (ATMQ-52), with four LIKE items and eleven ANX items. The pattern matrix showed a separation of the LIKE and IMP subscales and the ANX and LIKE subscales. The LIKE (without LIKE5) and ANX (without ANX2) scales had Cronbach alpha values of .82 and .77, respectively, indicating adequate internal consistency.

The ATMQ-52 (with LIKE5 and ANX2 removed) showed a simple structure consistent with Tag's (2005) original conception of the factor structure of the LIKE and ANX scales. Unlike the 54-item scale, all the items were loaded onto the correct subscales.

Table 2. *Pattern Matrix for PCA With Oblimin Rotation of the Ten-Factor Solution*

Scale/Item No.	Item	Component									
		1	2	3	4	5	6	7	8	9	10
CONF1+	Generally I have felt secure about attempting mathematics.	**.566**	-.045	-.058	.104	.104	.085	.002	-.092	-.065	.042
CONF2+	I am sure I could do advanced work in mathematics.	**.744**	.040	-.055	.019	.053	-.068	-.012	-.067	-.040	-.043
CONF3+	I am sure that I can learn mathematics.	**.722**	.009	-.002	-.084	-.071	.115	-.051	.007	-.086	.054
CONF4+	I think I could handle more difficult mathematics.	**.811**	.025	.079	.029	-.034	-.099	-.054	.050	.073	-.029
CONF5+	I can get good grades in mathematics.	**.615**	-.010	.085	.012	-.057	.145	-.001	.123	-.046	.197
CONF6+	I have a lot of self-confidence when it comes to math.	**.598**	-.006	.011	.189	.079	.014	-.038	-.048	-.017	.046
CONF7-	I'm no good at math.	.083	.054	-.039	.000	.095	-.005	.013	.032	-.038	**.718**
CONF8-	I don't think I could do advanced mathematics.	.116	.184	-.027	.001	-.040	-.054	.047	-.019	-.045	**.577**
CONF9-	I'm not the type to do well in math.	.101	.148	.026	-.031	-.047	.060	.011	.022	-.034	**.693**
CONF10-	For some reason even though I study, math seems unusually hard for me.	-.099	-.035	-.028	.276	.018	-.035	-.088	.012	.023	**.582**
CONF11-	Most subjects I can handle OK, but I have a knack for mucking up math.	.088	.075	.045	.009	-.019	.018	-.073	-.020	.020	**.725**
CONF12-	Math has been my worst subject.	-.033	-.028	.046	-.056	-.016	-.011	-.080	-.126	.019	**.776**
USE1+	I'll need mathematics for my future work.	-.079	.020	-.110	-.008	.033	-.012	.040	.018	**-.854**	.004
USE2+	I study mathematics because I know how useful it is.	.210	-.062	-.019	-.069	.090	.047	.002	-.125	**-.513**	.120
USE3+	Knowing mathematics will help me earn a living.	.073	.008	.190	-.054	-.028	.017	.004	.029	**-.635**	.025
USE4+	Mathematics is a worthwhile and necessary subject.	.020	.053	.186	-.022	-.089	.153	-.065	-.020	**-.596**	-.018
USE5+	I'll need a firm mastery of mathematics for my future work.	-.020	.120	.135	.097	-.006	-.050	.005	.016	**-.697**	-.073
USE6+	I will use mathematics in many ways as an adult.	.117	-.038	.084	.089	.018	-.046	-.059	.020	**-.540**	.003
USE7-	Mathematics is of no relevance to my life.	-.031	**.675**	.043	-.005	-.027	.041	.007	-.050	-.082	.075
USE8-	Mathematics will not be important to me in my life's work.	.025	**.714**	.001	.021	-.048	.054	.037	-.012	-.072	.001
USE9-	I see mathematics as a subject I will rarely use in daily life as an adult.	.054	**.701**	-.045	.044	.134	-.116	.062	.119	.041	.063
USE10-	Taking mathematics is a waste of time.	-.041	**.646**	.071	-.046	-.028	.122	-.104	-.122	-.014	.058
USE11-	In terms of my adult life it is not important for me to do well in math.	-.016	**.598**	.145	-.109	-.087	.167	-.099	-.054	.011	.115
USE12-	I expect to have little use for mathematics when I get out of school.	.035	**.617**	.132	-.014	.086	-.027	-.082	-.086	-.065	.050
IMP1+	I would like a job that involved using mathematics.	.087	.006	**.338**	.082	.180	-.254	-.002	-.211	-.143	.055
IMP2+	I need to do well in mathematics to get the job I want.	.058	.095	**.620**	.048	.130	-.164	.012	-.035	-.112	-.046
IMP3+	I need to do well in mathematics to get into the secondary school or university I prefer.	.055	.062	**.779**	-.001	.022	.066	.047	.056	.062	.051
IMP4+	Mathematics is important to everyone's life.	-.043	.012	**.689**	.039	.021	.044	-.046	.012	-.119	-.050
IMP5+	I think it is important to do well in mathematics at school.	.000	.075	**.653**	-.009	-.011	.017	-.066	.005	-.124	.043
LIKE1+	I like mathematics.	.163	-.049	.014	.049	.113	.055	-.054	**-.566**	-.167	.187

LIKE2+	I enjoy learning mathematics.	.153	-.002	.111	.008	.080	.114	-.040	**-.566**	-.102	.148
LIKE3-	Mathematics is boring.	-.050	.221	.111	-.007	.050	.058	-.229	**-.483**	.041	.167
LIKE4+	Mathematics is an easy subject.	.074	-.081	-.036	.332	.062	-.077	.049	**-.364**	-.110	.151
LIKE5+	I need to do well in mathematics to please myself.	.049	-.108	**.413**	-.016	-.017	.178	.040	-.231	-.166	.011
ANX1+	Math doesn't scare me at all.	.199	.036	-.129	**.539**	.048	-.102	-.002	-.174	-.105	-.031
ANX2+	It wouldn't bother me at all to take more math courses.	.109	.118	.051	.145	.159	.017	-.068	**-.393**	-.142	-.111
ANX3+	I haven't usually worried about being able to solve math problems.	.170	.014	.047	**.510**	.120	-.045	.026	-.114	-.050	.029
ANX4+	I almost never have got nervous during a math test.	-.020	.011	.006	**.835**	-.017	.005	-.016	.033	.017	-.029
ANX5+	I usually have been at ease during math tests.	.028	-.034	.028	**.825**	-.023	.034	.027	.038	.024	.040
ANX6+	I usually have been at ease in math classes.	.023	-.007	.114	**.620**	-.023	.194	-.025	.034	-.048	.061
ANX7-	Mathematics usually makes me feel uncomfortable and nervous.	.031	.207	.034	.026	-.046	.072	**-.516**	-.336	.038	.052
ANX8-	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	.053	.070	-.091	-.050	-.042	.015	**-.662**	-.146	-.038	-.100
ANX9-	I get a sinking feeling when I think of trying math problems.	.091	-.042	.119	-.003	.099	-.010	**-.640**	.187	.060	.054
ANX10-	My mind goes blank and I am unable to think clearly when working mathematics.	.015	-.075	.035	.001	.095	.021	**-.626**	.071	-.058	.224
ANX11-	A math test would scare me.	-.004	.051	-.064	.452	-.042	-.066	**-.494**	.057	-.013	.129
ANX12-	Mathematics makes me feel uneasy and confused.	.051	.070	.044	.123	-.007	-.047	**-.583**	-.165	-.024	.159
MATBEHA1+	I always do my math homework.	.098	-.068	.040	.027	.283	**.512**	.071	-.239	.013	.022
MATBEHA2+	I come to math classes on time.	.036	.044	.054	.056	.019	**.752**	.058	-.028	-.092	.013
MATBEHA3-	I skip math classes.	.023	.292	.016	.060	-.051	**.498**	-.173	.104	.023	-.082
MATBEHA4+	I often raise my hand in math classes.	.199	-.035	.014	.203	.237	**.272**	.084	-.092	.010	.257
TIME1-	I spend very little time on mathematics at home.	-.039	.096	-.111	-.057	**.521**	.182	-.204	.180	-.125	.108
TIME2+	I spend a lot of my study time on mathematics when compared to other subjects.	.043	.008	.086	.021	**.712**	-.005	.090	-.149	-.011	-.052
TIME3-	Compared to other subjects, I spend the least time on mathematics at home.	-.003	.214	-.076	-.082	**.531**	.097	-.204	.136	-.061	.146
TIME4+	I spend a lot of my time at home on mathematics.	.028	-.059	.170	.057	**.723**	-.057	.023	-.076	.028	-.071

Note. Negatively worded items are marked with a minus sign; positively worded items are marked with a plus sign. Major loadings for each item are bolded.

Table 3. *Structure Matrix for PCA With Oblimin Rotation of the Ten-Factor Solution*

Scale/Item No.	Item	Component									
		1	2	3	4	5	6	7	8	9	10
CONF1+	Generally I have felt secure about attempting mathematics.	**.700**	.078	.186	.383	.333	.189	-.142	-.322	-.350	.336
CONF2+	I am sure I could do advanced work in mathematics.	**.767**	.112	.187	.326	.283	.061	-.139	-.294	-.342	.285
CONF3+	I am sure that I can learn mathematics.	**.749**	.178	.241	.220	.182	.257	-.205	-.221	-.377	.356
CONF4+	I think I could handle more difficult mathematics.	**.774**	.107	.241	.323	.193	.032	-.171	-.182	-.263	.289
CONF5+	I can get good grades in mathematics.	**.706**	.189	.279	.286	.191	.282	-.200	-.127	-.344	.450
CONF6+	I have a lot of self-confidence when it comes to math.	**.746**	.119	.238	.472	.328	.136	-.200	-.304	-.347	.380
CONF7-	I'm no good at math.	.392	.328	.098	.250	.281	.157	-.309	-.132	-.210	**.783**
CONF8-	I don't think I could do advanced mathematics.	.354	.378	.112	.206	.144	.105	-.258	-.149	-.209	**.662**
CONF9-	I'm not the type to do well in math.	.387	.429	.168	.198	.163	.239	-.335	-.131	-.233	**.779**
CONF10-	For some reason even though I study, math seems unusually hard for me.	.227	.176	.022	.411	.163	.049	-.313	-.107	-.079	**.632**
CONF11-	Most subjects I can handle OK, but I have a knack for mucking up math.	.404	.385	.164	.262	.198	.196	-.402	-.176	-.198	**.820**
CONF12-	Math has been my worst subject.	.295	.290	.130	.192	.172	.144	-.361	-.233	-.152	**.784**
USE1+	I'll need mathematics for my future work.	.233	.169	.267	.110	.197	.104	-.030	-.183	**-.772**	.109
USE2+	I study mathematics because I know how useful it is.	.493	.143	.320	.187	.314	.189	-.102	-.352	**.648**	.293
USE3+	Knowing mathematics will help me earn a living.	.355	.213	.495	.112	.180	.170	-.088	-.209	**-.736**	.167
USE4+	Mathematics is a worthwhile and necessary subject.	.316	.281	.496	.109	.128	.301	-.168	-.234	**-.711**	.168
USE5+	I'll need a firm mastery of mathematics for my future work.	.307	.265	.471	.210	.196	.099	-.093	-.223	**-.770**	.117
USE6+	I will use mathematics in many ways as an adult.	.391	.133	.365	.258	.217	.078	-.131	-.213	**-.631**	.184
USE7-	Mathematics is of no relevance to my life.	.137	**.734**	.240	.019	.097	.231	-.298	-.119	-.272	.332
USE8-	Mathematics will not be important to me in my life's work.	.143	**.728**	.199	.022	.066	.230	-.260	-.076	-.248	.275
USE9-	I see mathematics as a subject I will rarely use in daily life as an adult.	.131	**.667**	.087	.053	.188	.051	-.233	.052	-.116	.305
USE10-	Taking mathematics is a waste of time.	.134	**.755**	.260	-.009	.103	.319	-.396	-.175	-.241	.348
USE11-	In terms of my adult life it is not important for me to do well in math.	.129	**.740**	.299	-.070	.044	.363	-.389	-.108	-.220	.365
USE12-	I expect to have little use for mathematics when I get out of school.	.249	**.724**	.348	.081	.239	.197	-.375	-.206	-.340	.369
IMP1+	I would like a job that involved using mathematics.	.368	.114	**.484**	.285	.347	-.113	-.088	-.402	-.426	.206
IMP2+	I need to do well in mathematics to get the job I want.	.306	.219	**.713**	.183	.284	-.003	-.077	-.252	-.462	.112
IMP3+	I need to do well in mathematics to get into the secondary school or university I prefer.	.261	.239	**.782**	.096	.171	.209	-.074	-.141	-.343	.156
IMP4+	Mathematics is important to everyone's life.	.207	.196	**.747**	.117	.168	.175	-.106	-.182	-.438	.072
IMP5+	I think it is important to do well in mathematics at school.	.259	.290	**.736**	.106	.163	.183	-.177	-.195	-.457	.185
LIKE1+	I like mathematics.	.541	.159	.316	.357	.382	.197	-.220	**-.735**	-.478	.427

LIKE2+	I enjoy learning mathematics.	.504	.202	.385	.290	.340	.259	-.210	**-.720**	-.452	.386
LIKE3-	Mathematics is boring.	.261	.426	.293	.174	.239	.225	-.423	**-.550**	-.255	.418
LIKE4+	Mathematics is an easy subject.	.395	-.014	.151	**.511**	.256	-.024	-.065	-.508	-.298	.302
LIKE5+	I need to do well in mathematics to please myself.	.298	.068	**.556**	.127	.163	.268	-.021	-.389	-.442	.111
ANX1+	Math doesn't scare me at all.	.460	.030	.073	**.663**	.240	-.061	-.115	-.355	-.277	.241
ANX2+	It wouldn't bother me at all to take more math courses.	.388	.203	.306	.317	.345	.131	-.177	**-.539**	-.408	.176
ANX3+	I haven't usually worried about being able to solve math problems.	.473	.059	.219	**.645**	.316	.016	-.116	-.330	-.300	.288
ANX4+	I almost never have got nervous during a math test.	.280	-.012	.079	**.808**	.129	-.018	-.121	-.140	-.124	.202
ANX5+	I usually have been at ease during math tests.	.342	-.035	.106	**.828**	.145	.015	-.098	-.157	-.143	.257
ANX6+	I usually have been at ease in math classes.	.357	.097	.241	**.657**	.169	.223	-.179	-.174	-.258	.300
ANX7-	Mathematics usually makes me feel uncomfortable and nervous.	.268	.460	.203	.186	.140	.240	**-.654**	-.398	-.201	.400
ANX8-	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	.140	.295	.023	.057	.055	.129	**-.656**	-.164	-.104	.201
ANX9-	I get a sinking feeling when I think of trying math problems.	.195	.249	.137	.119	.178	.116	**-.662**	.082	-.063	.312
ANX10-	My mind goes blank and I am unable to think clearly when working mathematics.	.245	.285	.129	.179	.227	.170	**-.705**	-.053	-.172	.473
ANX11-	A math test would scare me.	.262	.249	.026	.532	.114	.026	**-.604**	-.075	-.113	.431
ANX12-	Mathematics makes me feel uneasy and confused.	.331	.372	.193	.317	.190	.124	**-.707**	-.289	-.229	.491
MATBEHA1+	I always do my math homework.	.340	.097	.237	.166	.416	**.556**	-.069	-.372	-.260	.213
MATBEHA2+	I come to math classes on time.	.244	.246	.254	.087	.164	**.789**	-.121	-.152	-.290	.205
MATBEHA3-	I skip math classes.	.108	.437	.137	.022	.034	**.569**	-.336	.045	-.116	.177
MATBEHA4+	I often raise my hand in math classes.	**.502**	.145	.209	.406	.430	.358	-.145	-.300	-.280	.465
TIME1-	I spend very little time on mathematics at home.	.180	.307	.061	.057	**.553**	.299	-.353	.037	-.219	.322
TIME2+	I spend a lot of my study time on mathematics when compared to other subjects.	.302	.067	.256	.196	**.756**	.085	-.007	-.322	-.279	.133
TIME3-	Compared to other subjects, I spend the least time on mathematics at home.	.222	.420	.101	.060	**.580**	.255	-.403	-.009	-.216	.396
TIME4+	I spend a lot of my time at home on mathematics.	.277	.011	.287	.220	**.750**	.024	-.035	-.256	-.240	.103

Note. Negatively worded items are marked with a minus sign; positively worded items are marked with a plus sign. Major loadings for each item are bolded.

3.4. Confirmatory Factor Analysis

A CFA using maximum likelihood estimation was conducted on a sample of 1960 7th-grade students. An alternative model was also investigated. A 52-item, ten-factor model, as identified in the PCA, was investigated, allowing the factors to freely correlate. Factor loadings in this model were statistically significant. Although the chi-square test was significant [chi-square (1229) = 8834.59, p = .0], the other fit indices, the CFI (.97), NNFI (.97), SRMR (.066), and RMSEA (.056) indicated a good fit. Lambda-x estimates, which are analogous to factor loadings in exploratory factor analysis, for the latent factors for ATMQ are presented in Table 4.

Table 4. *Lambda-X Estimates for ATMQ*

	Indicator	Lambda-x
Confidence in Learning Mathematics-Positive Items (CONF_P)	CONF1	.70
	CONF2	.69
	CONF3	.68
	CONF4	.66
	CONF5	.67
	CONF6	.76
Confidence in Learning Mathematics-Negative Items (CONF_N)	CONF7	.74
	CONF8	.63
	CONF9	.77
	CONF10	.55
	CONF11	.82
	CONF12	.72
Usefulness of Mathematics-Positive Items (USE_P)	USE1	.60
	USE2	.64
	USE3	.71
	USE4	.70
	USE5	.71
	USE6	.59
Usefulness of Mathematics-Negative Items (USE_N)	USE7	.69
	USE8	.64
	USE9	.52
	USE10	.76
	USE11	.76
	USE12	.73
Importance of Mathematics (IMP)	IMP1	.55
	IMP2	.70
	IMP3	.66
	IMP4	.67
	IMP5	.69
Liking of Mathematics (LIKE)	LIKE1	.88
	LIKE2	.84
	LIKE3	.57
	LIKE4	.53
Mathematics Anxiety-Positive Items (ANX_P)	ANX1	.67
	ANX3	.67
	ANX4	.69
	ANX5	.74
	ANX6	.62
Mathematics Anxiety-Negative Items (ANX_N)	ANX7	.69
	ANX8	.48
	ANX9	.50
	ANX10	.65
	ANX11	.65
	ANX12	.79
Learner Behaviors toward Mathematics (MATBEHA)	MATBEHA1	.60
	MATBEHA2	.52
	MATBEHA3	.31
	MATBEHA4	.69
Time Spent on Mathematics at Home (TIME)	TIME1	.37
	TIME2	.76
	TIME3	.41
	TIME4	.72

Phi values are the estimates for the covariances between the latent constructs, analogous to the scale correlations in exploratory factor analysis given in Table 5. Note that the values presented in Tables 4 and 5 are analogous to, but not equivalents of, factor loadings and scale correlations, respectively.

Table 5. *Phi Estimates*

	CONF_P	CONF_N	USE_P	USE_N	IMP	LIKE	ANX_P	ANX_N	MATBEHA
CONF_N	.61								
USE_P	.61	.36							
USE_N	.31	.60	.47						
IMP	.50	.31	.81	.48					
LIKE	.72	.57	.64	.40	.58				
ANX_P	.66	.45	.38	.09	.34	.57			
ANX_N	.52	.73	.35	.64	.34	.55	.46		
MATBEHA	.72	.60	.58	.49	.53	.72	.50	.52	
TIME	.51	.35	.48	.28	.48	.57	.39	.34	.60

Alternatively, the full 54-item original version of the ATMQ model was tested. Fit statistics for the ten-factor model for all 54 items showed an acceptable fit to the data. The chi-square test was significant [chi-square (1332) = 10031.28, p = .0]. The CFI (.97), NNFI (.96), SRMR (.069), and RMSEA (.058) fit statistics were adequate but not better than those obtained from the 52-item ten-factor solution.

Overall, the best fitting model for this data was a 52-item ten-correlated factor model representing CONF_P, CONF_N, USE_P, USE_N, IMP, LIKE, ANX_P, ANX_N, MATBEHA, and TIME, but with LIKE5 and ANX2 removed from the respective scales.

3.5. Second-Order Confirmatory Factor Analysis

Following the general sequence of CFA-based higher-order factor analysis (Brown, 2015), the first step was to fit a three-factor CFA model, allowing the correlations among the factors to be freely estimated. Considering the factor structure indicated in Table 4, latent variables were formed for the three-factor CFA model. In establishing the measurement models, it is recommended to identify three to four indicators of each latent variable (Schumacher & Lomax, 2010). Therefore, the questionnaire items CONF6, CONF11, USE7, and IMP2 were used to define the cognitive component of attitude toward mathematics, labeled "cognitive". Items LIKE1, LIKE2, and ANX5 were used to define the affective component of attitude toward mathematics, labeled "affective". The last factor in the model, the behavioral component of attitude toward mathematics, labeled "behavioral", included the items MATBEHA1, MATBEHA4, TIME1, and TIME4.

The three-factor solution provided a poor fit to the data. The chi-square test was significant [chi-square (41) = 564.02, p = .0]. The CFI (.95), NNFI (.93), and SRMR (.051) fit statistics fulfilled the recommended guidelines, but the RMSEA (.081) statistic was barely adequate. Inspection of the residuals and modification indices revealed some items falling into correlation. The final fit statistics showed a good fit to the data. Although the chi-square statistic was significant [chi-square (36) = 245.06, p = .00], the CFI (.98) and NNFI (.97), SRMR (.036), and RMSEA (.054) indicated a good fit. The completely standardized parameter estimates of this solution are presented in Figure 1. As seen in the figure, all 11 items are indicators of their respective factors.

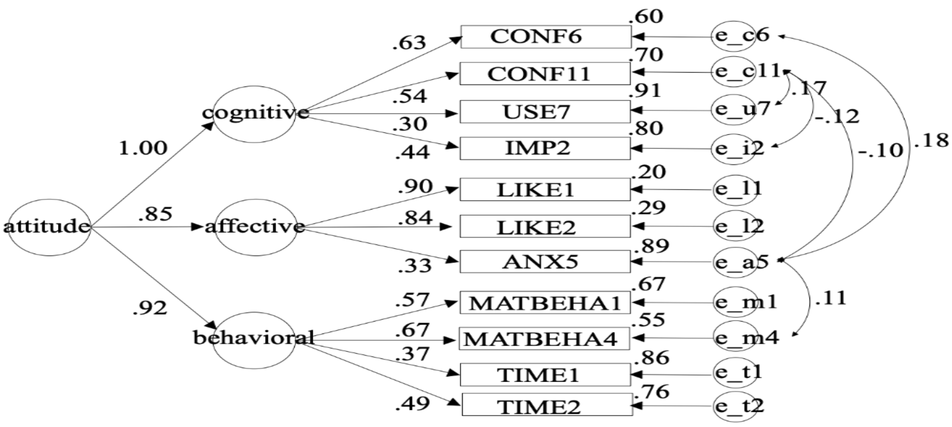

Figure 1. *Second-Order Confirmatory Factor Model of Attitude Toward Mathematics*

The results from the phi matrix in Table 6 provide the correlations among the factors. Consistent with prediction, all factors were significantly interrelated and were roughly the same, favoring a single second-order factor, "attitude".

Table 6. *Phi Estimates, Z-Scores, and Standard Errors for the CFA*

First-order Factor		Phi	*z*-score	*SE*
Cognitive	Affective	0.84	44.58	0.02
	Behavioral	0.90	37.35	0.02
Affective	Behavioral	0.79	42.55	0.02

From Figure 1, it is also seen that each of the first-order factors loaded strongly onto the second-order factor (range of loadings = 1.00–.85). The value of 1.00 for the cognitive was not problematic. It is indicated that if the factors are *correlated*, the factor loadings are *regression coefficients* and *not correlations*, and as such, they can be larger than one in magnitude (Jöreskog, 1999).

The estimates provided in the psi matrix indicate the proportion of variance in the first-order factors that is not explained by the second-order factor given in Table 7. These estimates indicated that the second-order factor accounted for 99–72% of the variance in the first-order factors.

Table 7. *Psi Estimates, Z-Scores, and Standard Errors for the Second-Order CFA*

First-order Factor	Psi	*z*-score	*SE*
Cognitive	.01	0.03	.04
Affective	.28	10.93	.03
Behavioral	.15	4.12	.04

Therefore, this study concludes that the measure of students' attitudes toward mathematics is predominantly a function of the cognitive variable, with complementary affective and behavioral components enhancing overall attitude. We also found that the cognitive component is indicated as a stronger measure of attitude, followed by the behavioral and affective components.

4. Conclusion, Discussion and Recommendations

In the present study, we presented a factor model demonstrating the second-order factor structure of attitude toward mathematics: cognitive, affective, and behavioral. Before discussing the results of the model obtained, it is useful to discuss the results of the respective descriptive analysis. The results of the descriptive analysis showed that the students surveyed in this study were likely to have positive attitudes toward mathematics, as measured by the scales of the ATMQ. This is supported by the findings of previous studies showing that Turkish students generally had positive attitudes toward mathematics classes (Sen & Koca, 2005; Unlu, 2007).

In the main study, the model showed that cognitive, affective, and behavioral components explain attitudes toward mathematics. In mathematics attitude research, it is claimed that attitudes have typically been defined as consisting of cognitive (beliefs), affective (emotions), and conative (behavior) dimensions (Hannula, 2011). The model hypothesized for the present study is exactly what Malmivuori (2001) stated: attitude would constitute a single second-order factor, while beliefs, affective responses, and related behavior alone would operate as first-order affective factors. We, therefore, hypothesized a second-order factor model for the construct attitude for this study.

During the specification search in the *a priori* model, error covariances between the observed variables were added. This is supported by recognizing the intersection of the domains of the FSMAS (Fennema & Sherman, 1976).

In the present study, six attitudinal variables identified the three components of attitude toward mathematics. This was anticipated after reviewing the literature regarding the definition of each component. Moreover, Hannula (2011) classified the subscales of FSMAS into some dimensions of mathematics-related affect (beliefs, emotional traits, and motivations). This study measured the strength of the relationships between the three components and attitude itself. It was determined that the cognitive component of attitude provided the strongest measure of attitude. As Hannula (2011) indicated, beliefs influence the initiation of emotions, and repeated emotional reactions are the origin of attitudes. Therefore, as expected, in this study students' beliefs about mathematics and themselves played an important role in developing their attitudes toward mathematics.

Moreover, strong relationships between the three components of attitude were recognized. This is quite similar to the argument of Triandis (1971) that the three components are generally closely related. This study also found that the cognitive component was more related to the affective and behavioral components than the affective and behavioral components. From the view of the synergistic relations between the three categories of mathematics-related affect (cognition, emotion, and motivation), in his framework, Hannula (2011) pointed out that cognition codes the personal information about self and the environment; motivation gives direction for behavior; success or failure in motivation-directed behavior is reflected in emotions; and these emotions, in turn, can influence cognition.

In summary, in the present study, we have attempted to explain the structural and dynamic nature of attitude, which is probably the most problematic construct of mathematics-related affect. As Hannula (2011) claimed, the refinement of the conceptual framework and the focus on the dynamic nature of affect have highlighted the need to explore the structural properties of affect. Therefore, we tested the tripartite model of attitude in this study and investigated the synergic relationships between its components.

Based on both the findings of this study and the related studies in the literature, some recommendations can be given for future research. As proposed by Hannula (2006), there is a need to go beyond the simplistic positive-negative attitude distinction. Many of the mathematics attitude scales constructed in research studies are generally intended to assess factors such as liking or disliking, usefulness, and confidence. The choice of using items only about beliefs or emotions does not consider behaviors. As regards the multidimensional definition of attitude, the behavioral component should also be considered. Moreover, in research regarding attitude, we agree that there is a need to use several data collection tools, such as observations and interviews, as well as questionnaires for triangulation.

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